Star Formation

NCTS-Theoretical and Computational Astrophysics Summer Student Program Workshop 2021

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Outline

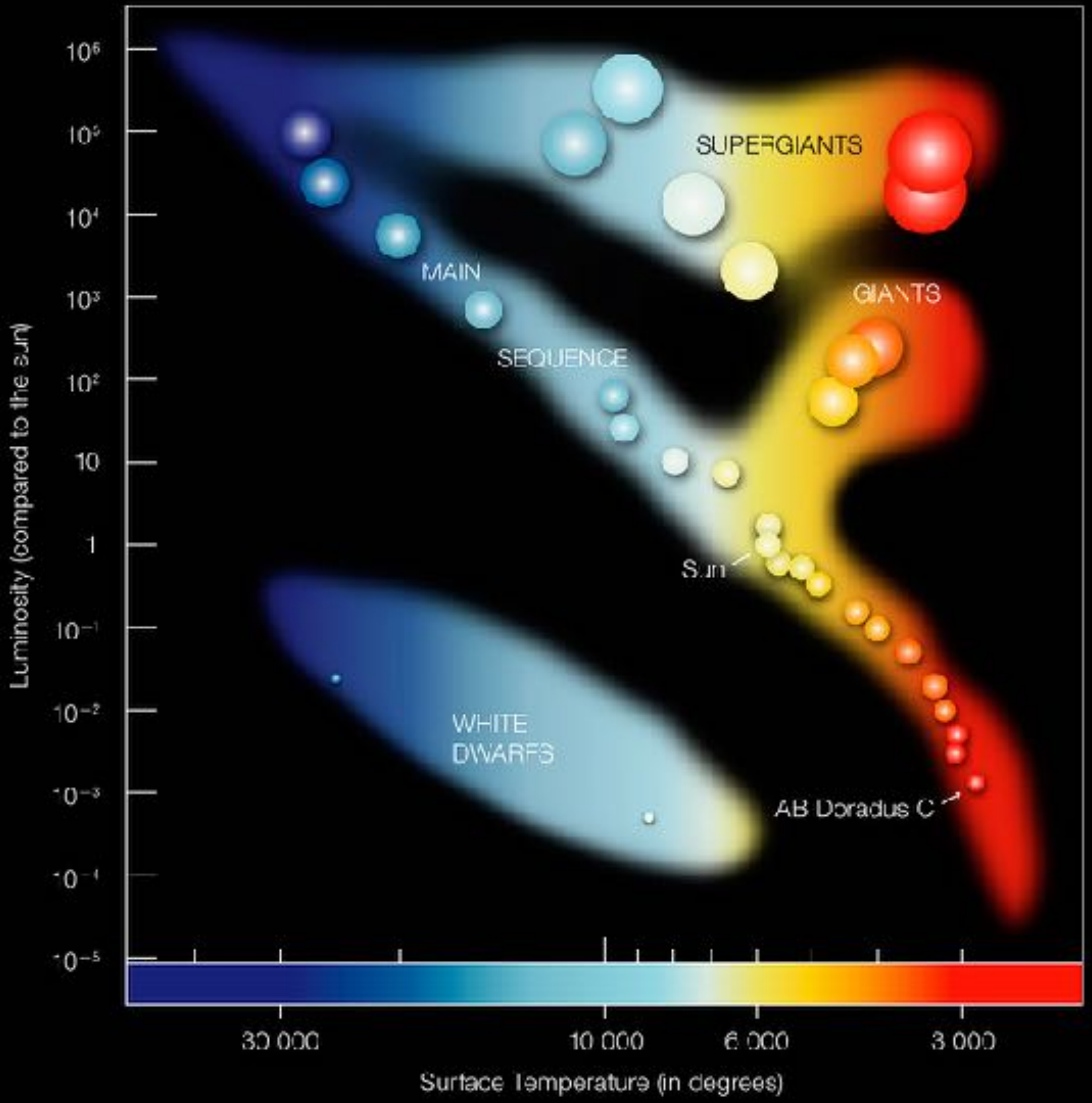
Star Formation in various contexts

- Which characteristics describe a star?
- What determines the properties of a star?
- Star formation in stellar clusters

What characteristics describe a star?

Try to name a few!

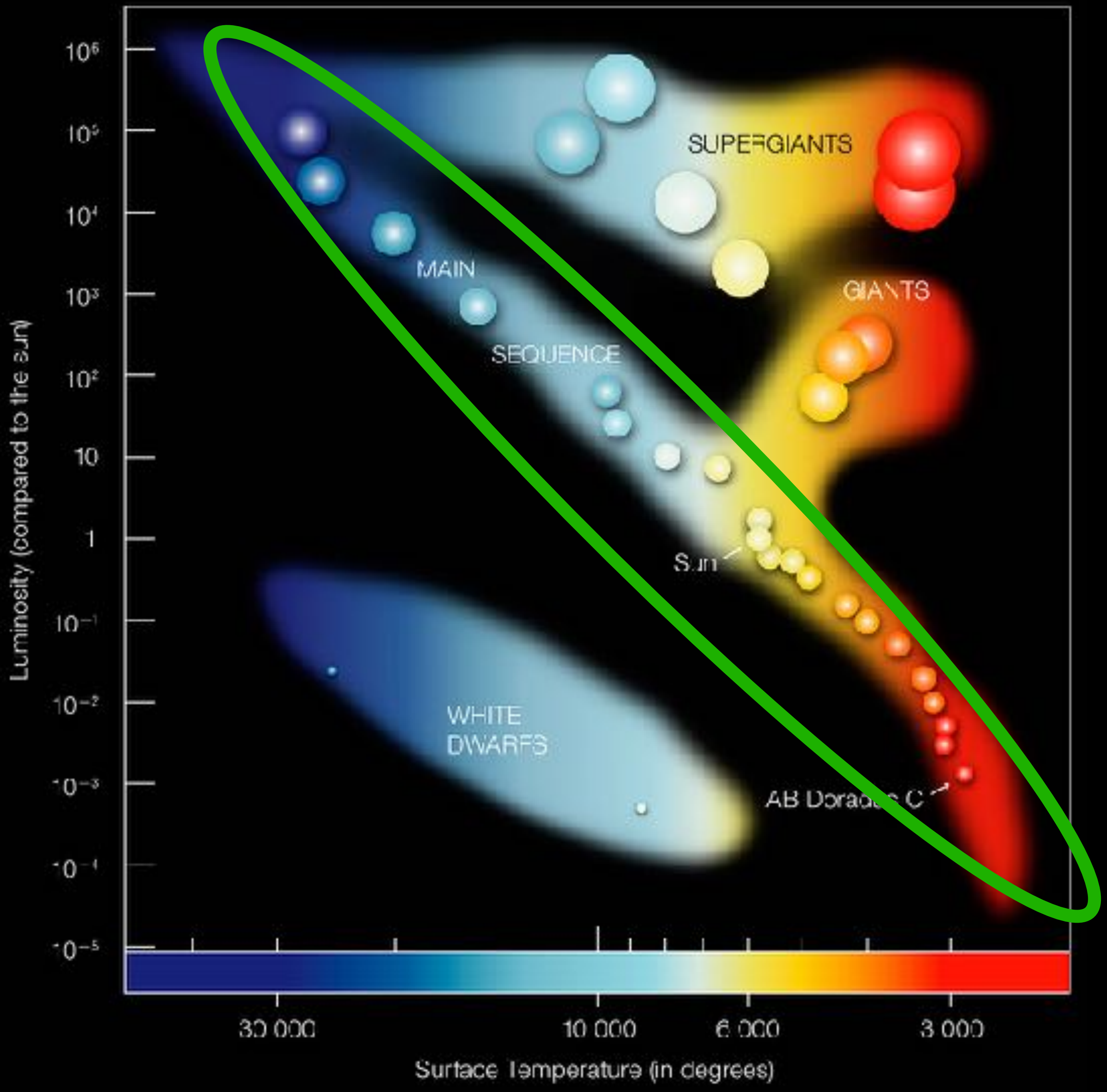
- Mass
- Color
- Temperature
- Size
- Metallicity
- Rotation velocity
- Luminosity
- Magnetic field



Hertzsprung-Russell Diagram (from eco.org)

What characteristics describe a star?

Mass is the most important!



Hertzsprung-Russell Diagram (from eco.org)

The **main sequence stars** (90 % of stars in the Universe) can be parametrized with almost one single parameter: the **mass**

What determines the properties of a star?

Governing factors

- Molecular gas H_2 (, CO, H_2O, …), $T \sim 10\ \mathrm{K}$, $n \sim 10^4\ \mathrm{cm}^{-3}$
- 1 % of mass in dust (metals)
- Collapse
  - Gravity
  - Shock events triggered compression (supernova blast wave, cloud-cloud colliding, nearby massive star formation, spiral arm shock in a galaxy, turbulence)
- Support
  - Thermal pressure (microscopic kinetic energy)
  - Turbulence (macroscopic kinetic energy)
  - Magnetic field (magnetic pressure)

What determines the properties of a star?

Jeans fragmentation: the critical length and mass

- Compare characteristic **timescales**: gravity vs thermal pressure
- Timescale of gravity: free-fall time

$$\ddot{R} = \frac{d^2R}{dt^2} = -\frac{GM}{R^2}$$

$$\tau_{\mathrm{ff}} = \int_{R=R_{\mathrm{c}}}^{R=0} dt = \sqrt{\frac{3\pi}{32G\rho}}$$

- Timescale of thermal pressure wave propagation: sound-crossing time

$$c_{\mathrm{s}}^2 = \frac{\partial P}{\partial \rho} = \frac{P}{\rho}\bigg|_{\mathrm{isothermal}} = \frac{kT}{\mu m_{\mathrm{p}}}, \quad (k \text{ Boltzmann constant}, \mu \sim 2.3 \text{ mean molecular weight}, m_{\mathrm{p}} \text{ proton mass})$$

$$\tau_{\mathrm{ss}} = \frac{R}{c_{\mathrm{s}}} = R\sqrt{\frac{\mu m_{\mathrm{p}}}{kT}}$$

- Jeans length $\lambda_{\mathrm{J}} = 2R_{\mathrm{crit}} = \sqrt{\frac{3\pi kT}{8G\mu m_{\mathrm{p}}\rho}} \approx 0.20 \text{ pc} \left(\frac{T}{10 \text{ K}}\right)^{0.5} \left(\frac{n}{10^4 \text{ cm}^{-3}}\right)^{-0.5}$
- Jeans mass $M_{\mathrm{J}} = \frac{4\pi}{3}\rho R_{\mathrm{crit}}^3 = \sqrt{\frac{3\pi^5 k^3 T^3}{2048 G^3 \mu^3 m_{\mathrm{p}}^3 \rho}} \approx 1.0\ M_{\odot} \left(\frac{T}{10 \text{ K}}\right)^{1.5} \left(\frac{n}{10^4 \text{ cm}^{-3}}\right)^{-0.5}$

What determines the properties of a star?

Jeans fragmentation: the critical length and mass

- Compare characteristic **energies**: gravity vs thermal pressure
- Gravitational energy

$$E_{\text{gravitational}} = \frac{3GM^2}{5R} = \frac{16\pi^2 G\rho^2 R^5}{15}$$

- Timescale of thermal pressure wave propagation: sound-crossing time

$$E_{\text{thermal}} = \frac{3}{2}PV = \frac{3}{2}\frac{\rho kT}{\mu m_{\text{p}}}\frac{4\pi}{3}R^3$$

- Jeans length $\lambda_{\text{J}} = 2R_{\text{crit}} = \sqrt{\frac{15kT}{2\pi G\mu m_{\text{p}}\rho}} \approx 0.30\ \text{pc}\left(\frac{T}{10\ \text{K}}\right)^{0.5}\left(\frac{n}{10^4\ \text{cm}^{-3}}\right)^{-0.5}$
- Jeans mass $M_{\text{J}} = \frac{4\pi}{3}\rho R_{\text{crit}}^3 = \sqrt{\frac{375k^3T^3}{32\pi G^3\mu^3 m_{\text{p}}^3\rho}} \approx 3.0\ M_\odot\left(\frac{T}{10\ \text{K}}\right)^{1.5}\left(\frac{n}{10^4\ \text{cm}^{-3}}\right)^{-0.5}$

What determines the properties of a star?

Jeans fragmentation: the critical length and mass

- Jeans length and Jeans mass from timescale or energy arguments only differ by a factor of a few, which is not of concern is geometric factors are also considered.
- Jeans length $\lambda_\mathrm{J} \propto T^{0.5}\rho^{-0.5}$
- Jeans mass $M_\mathrm{J} \propto T^{1.5}\rho^{-0.5}$
- Perturbative analysis of the Navier-Stokes equations will also give similar results

What determines the properties of a star?

The collapse: criterion for collapse

- Jeans mass gives the typical mass of a fragment, and what happens then?
- Consider a gas clump $M \approx \rho R^3$
- The equation of state is an effective polytrope $P = K\rho^\gamma$

$$PV \approx \frac{GM^2}{R} \left(\approx K\rho^\gamma \frac{M}{\rho} \approx GM^2 \left(\frac{\rho}{M}\right)^{\frac{1}{3}} \right)$$

- Energy ratio $\dfrac{E_{\text{thermal}}}{-E_{\text{gravitational}}} \approx KM^{-\frac{2}{3}}\rho^{\gamma-4/3}$
- At thermal equilibrium, $E_{\text{thermal}} \approx -E_{\text{gravitational}}$
- What happens if the clump contracts, i.e., density increases?

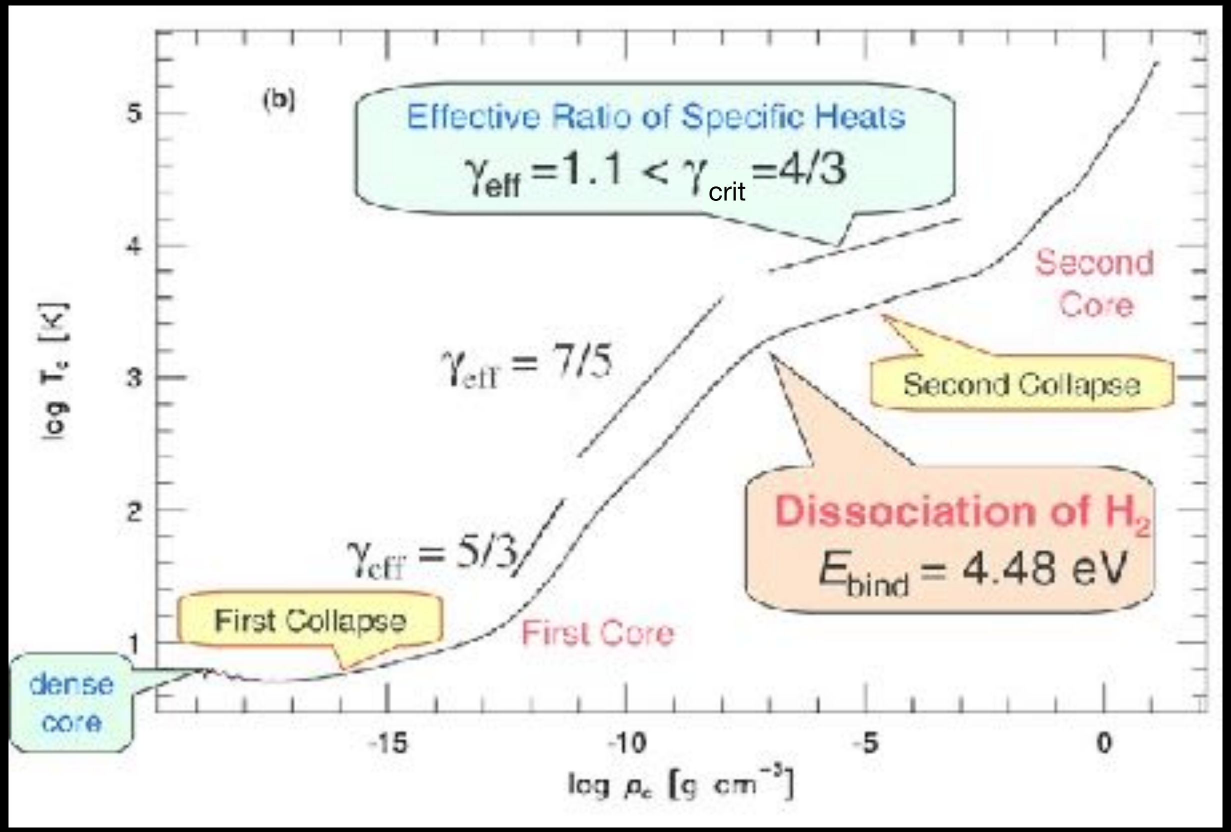
Critical value of gamma $\gamma_{\text{crit}} = \dfrac{4}{3}$, collapse possible only if $\gamma < \gamma_{\text{crit}}$

What determines the properties of a star?

The collapse: loss of energy

- Collapse means release of gravitational energy

  -> thermal, kinetic, radiation energies
- Polytropic equation of state (eos) $P = K\rho^\gamma$
- For adiabatic mono-atomic gas $\gamma_{\text{adiabatic}} = C_P/C_V = 5/3$
- The star forming gas is mainly molecular hydrogen (H_2) with some other molecules
- At molecular cloud densities, infrared radiation from de-excitation lines can escape easily: $\gamma < \gamma_{\text{adiabatic}}$



Inutsuka 2012

- Several regimes:
  - Prestellar core $\rho \lesssim 10^{-14}\ \text{g cm}^{-3}$, optically thin, $\gamma \approx 1$
  - First Hydrostatitc Core (FHC) $\rho \gtrsim 10^{-14}\ \text{cm}^{-3}$, dust heating, $\gamma \approx 5/3$
  - Hydrogen rotational levels excited $T \gtrsim 100\ \text{K}$, $\gamma \approx 7/5$
  - Second collapse $T \gtrsim 1000\ \text{K}$, Hydrogen dissociation, $\gamma \approx 1.1$
  - Second Hydrostatic Core (protostar) $T \gtrsim 2000\ \text{K}, \rho \gtrsim 10^{-2}\ \text{g cm}^{-3}$, $\gamma \approx 5/3$

What determines the properties of a star?

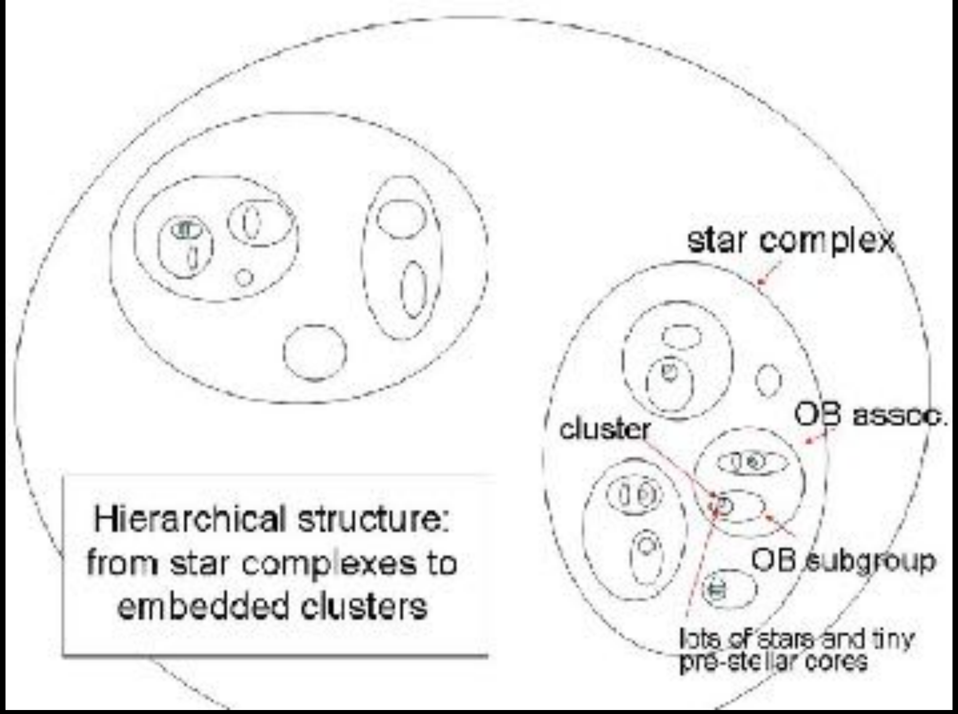
The collapse: reaching a new balance

- Hydrostatic equilibrium is possible only when radiative energy loss in inefficient ($\gamma > 4/3$)
- Force balance $\frac{1}{\rho}\frac{dP}{dr} = g(r) = -\frac{GM(r)}{r^2}$
- Definition of star: H nuclear fusion at central temperature $T \sim 10^7$ K
- Stellar mass range $0.08 - 100\ M_\odot$

What determines the properties of a star?

Does 1 core form 1 star?

- Recall the Jeans mass: $M_{\rm J} \propto T^{1.5}\rho^{-0.5}$
- What happens when a clump with $1\ M_{\rm J}$ collapses?
- Collapse usually leads to fragmentation, such that structure formation is hierarchical
    - Molecular cloud
    - Cloud cores/dense clumps
    - Stellar proto-clusters
    - Prestellar cores
    - Multiple star systems
- Fragmentation stops when heating stabilizes the collapse



Elmegreen 2014

What determines the properties of a star?

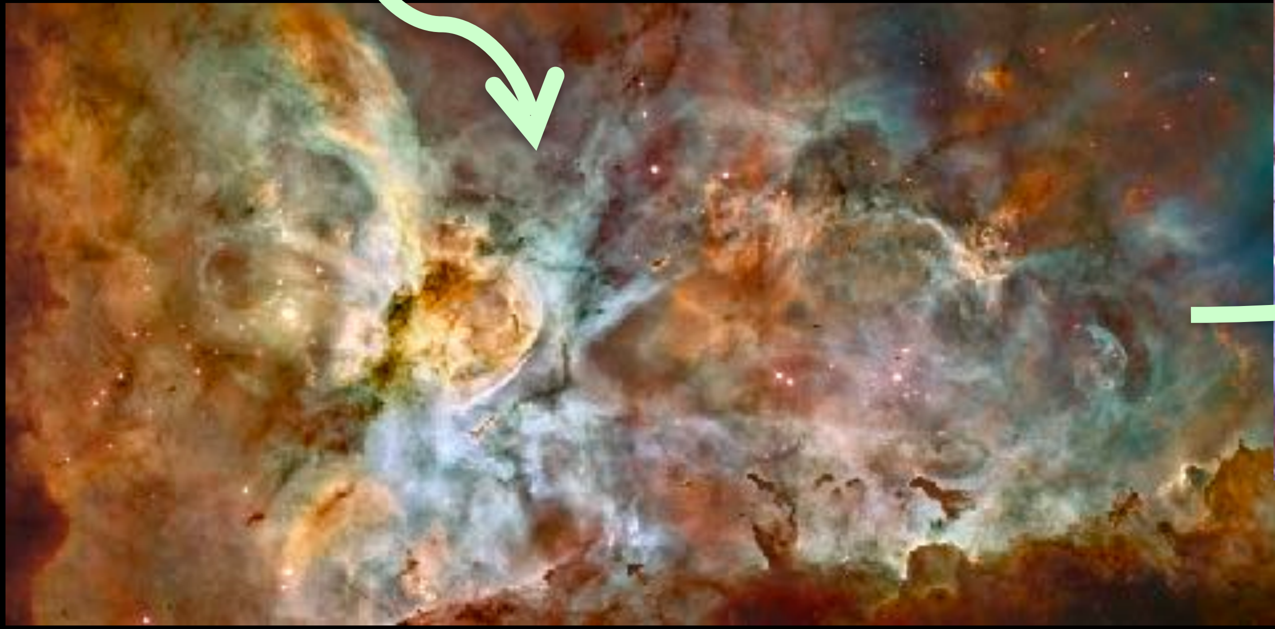
Mass accretion and stellar feedback

- Star formation is self-regulated by feedback, at stellar and cluster scales
- Early-type feedback: M_J can be locally increased by heating, while outflow can prevent further accretion
    - Protostars radiate in infrared
    - Protostellar jets inject momentum
- Late-type feedback: stop star formation activity by gas expulsion (3-4 Myr)
    - Supernovae inject momentum and create shocks
    - Stellar winds from massive star blow away star-forming gas

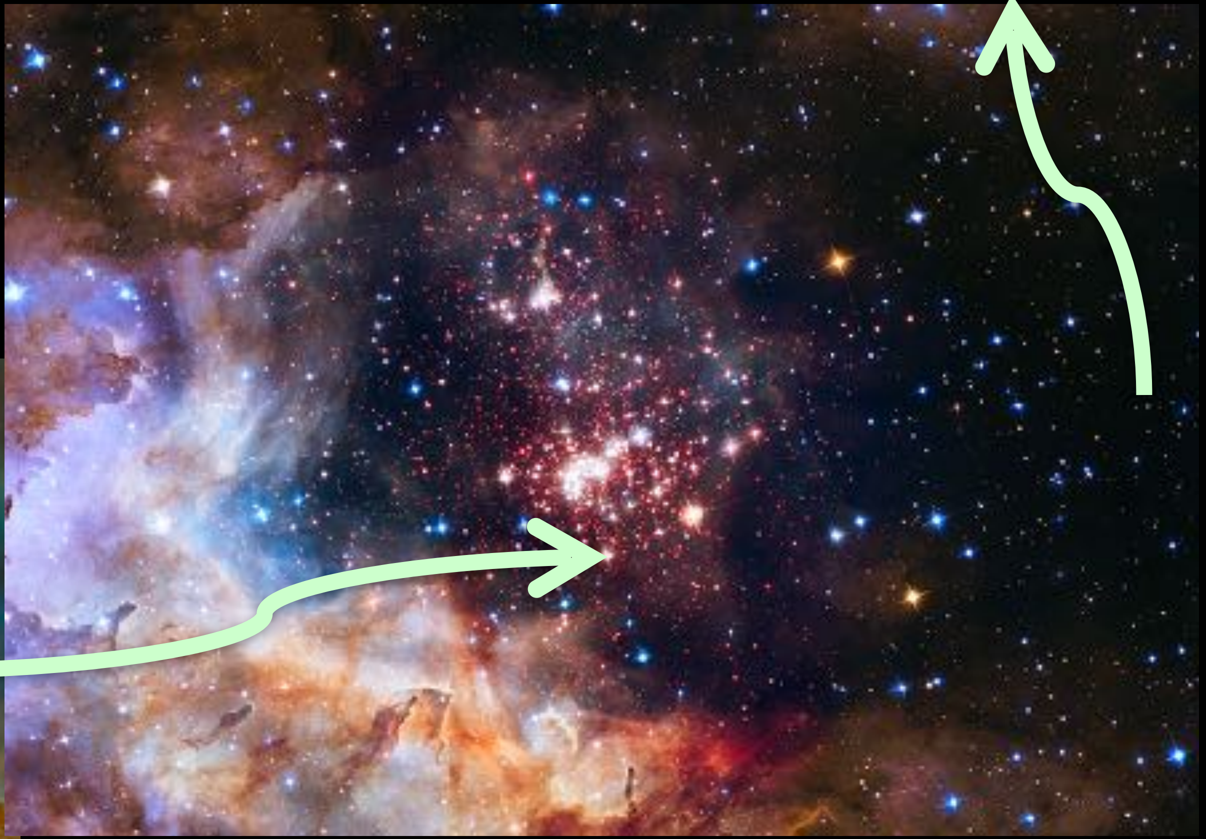
Star formation in clusters

The interstellar cycle

Galaxy ~kpc - Mpc



Molecular cloud ~ 100 pc



Stellar cluster ~ 1pc

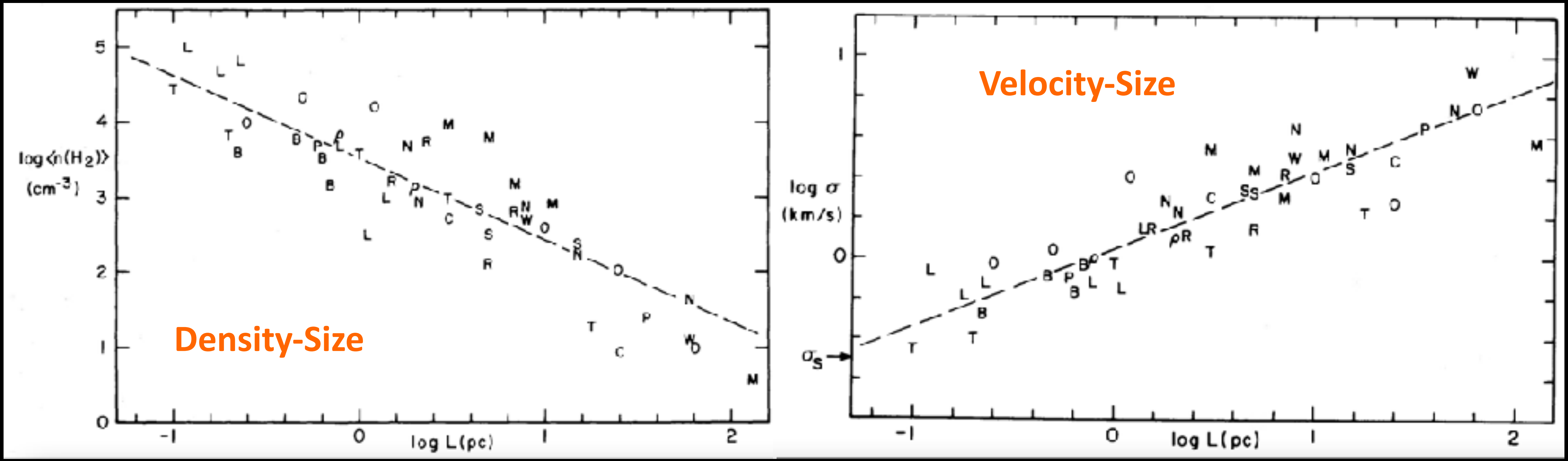
Stars ~ AU ~ light hrs

1 pc ~ 3 light years ~ 3 10^{16} m
1 AU ~ 5 10^{-6} pc ~ 0.1 light hour

Star formation in stellar clusters

Properties of the star-forming gas: the Larson relations

Scaling relations for the mass, size, and velocity dispersion of molecular clouds



Larson 1981

$$n = 3400\ \mathrm{cm}^{-3}\left(\frac{L}{1\ \mathrm{pc}}\right)^{-\eta},\ \eta = 1.1$$

$$\sigma = 1.1\ \mathrm{km\ s}^{-1}\left(\frac{L}{1\ \mathrm{pc}}\right)^{\eta_{\mathrm{turb}}},\ \eta_{\mathrm{turb}} = 0.38$$

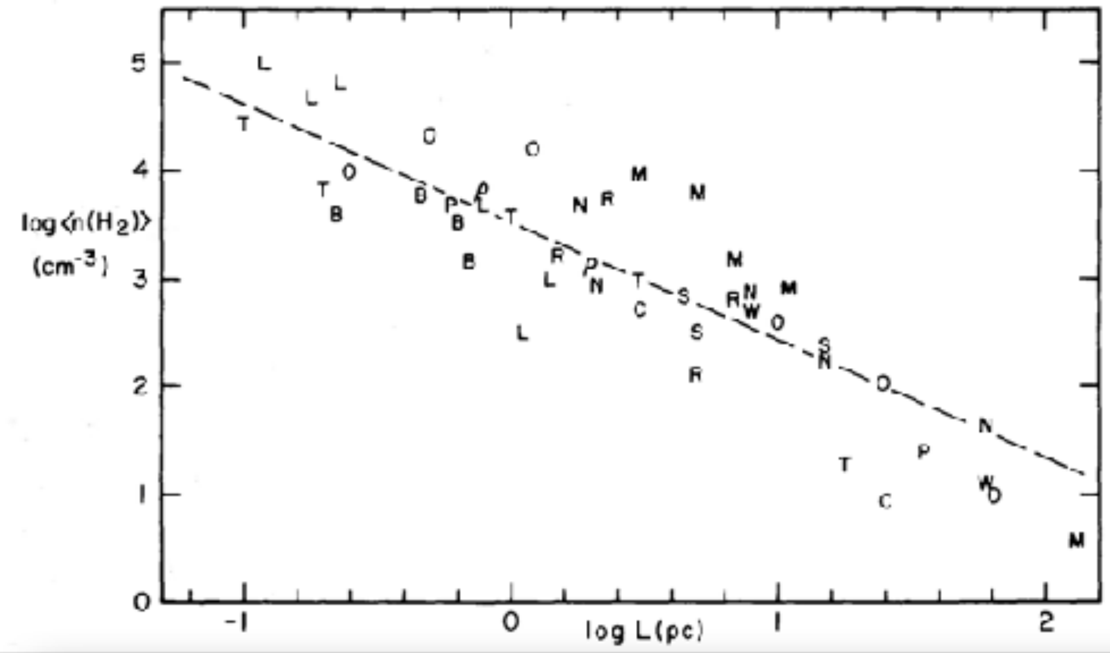
Star formation in stellar clusters

Properties of the star-forming gas: the density-size relation implies a fractal structure for the

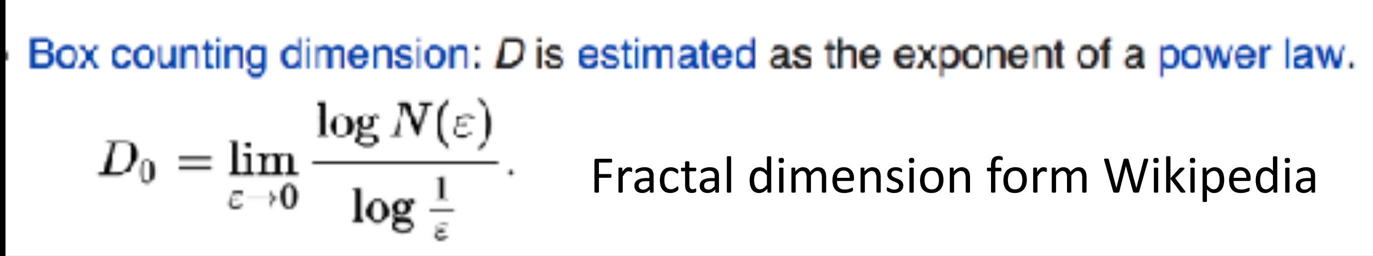
- Larson relation

$$n = 3400\ \mathrm{cm}^{-3} \left(\frac{L}{1\ \mathrm{pc}}\right)^{-\eta},\ \eta = 1.1$$

- $M \propto nL^3 \propto L^{3-\eta} = L^{1.9}$
- The interstellar space is not uniformly filled!
- The interstellar medium (ISM) is a fractal



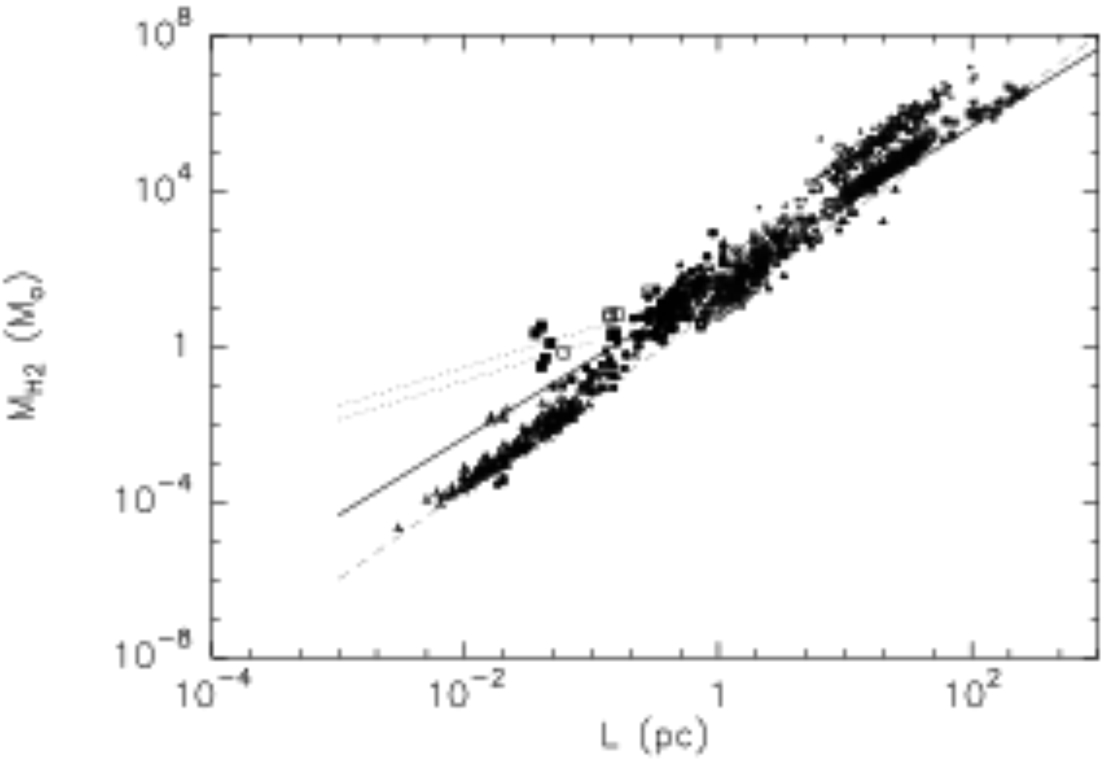
Larson 1981



Box counting dimension: D is estimated as the exponent of a power law.

$$D_0 = \lim_{\varepsilon \to 0} \frac{\log N(\varepsilon)}{\log \frac{1}{\varepsilon}}.$$

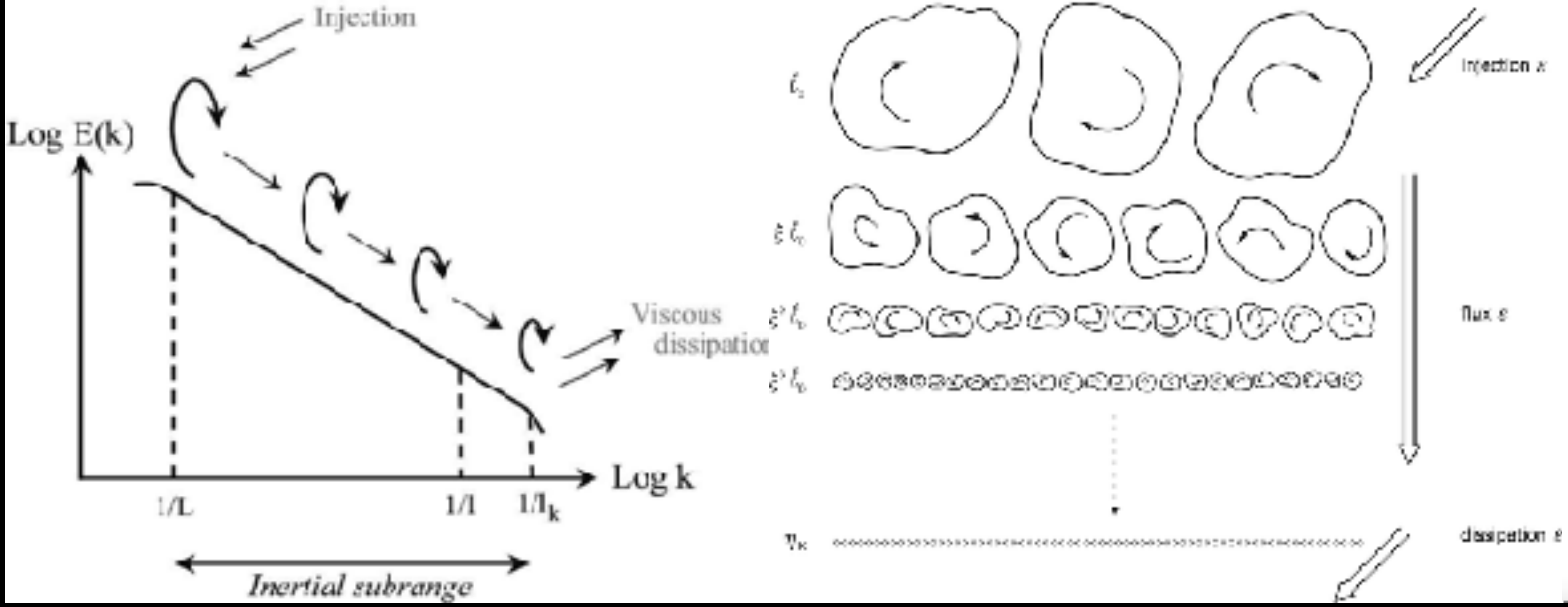
Fractal dimension form Wikipedia



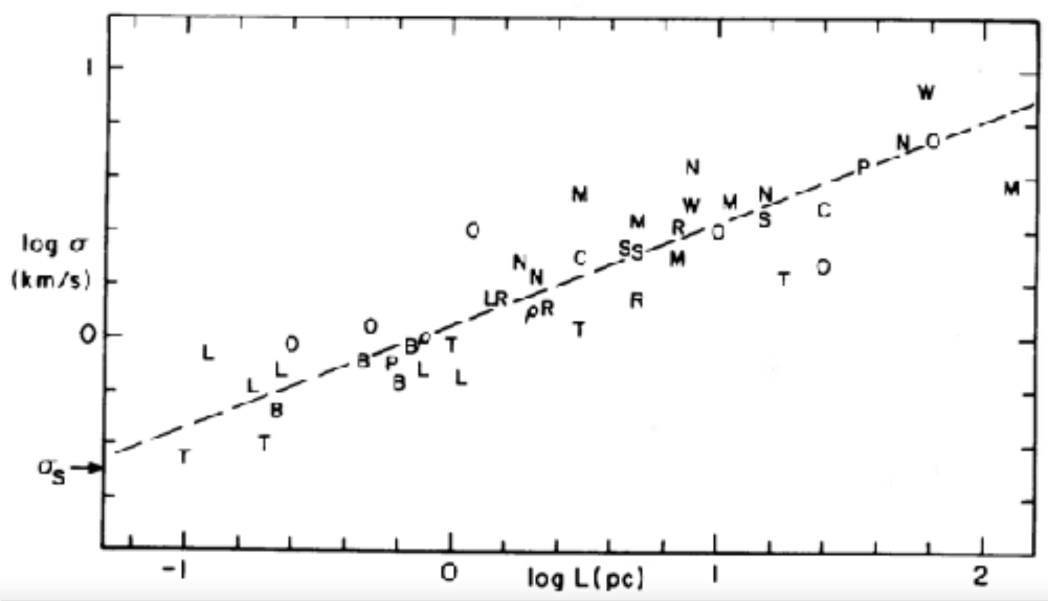
Hennebelle & Falgarone 2012

Star formation in stellar clusters

Properties of the star-forming gas: the turbulent velocity scaling



The Kolmogorov energy cascade

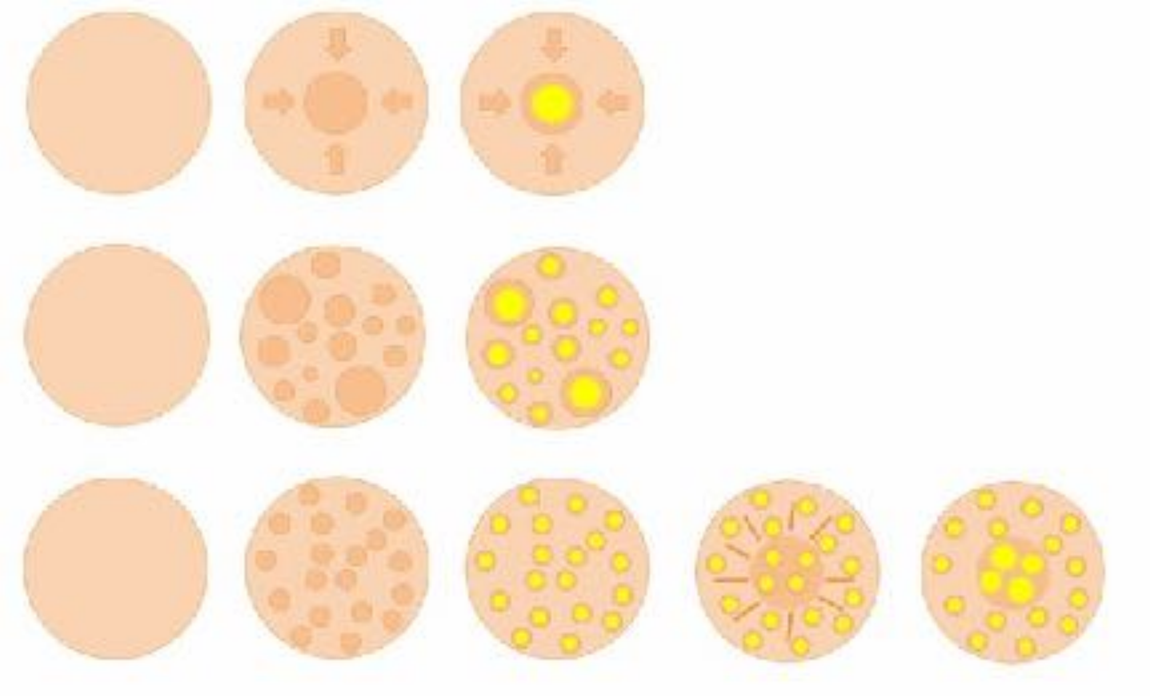


- Larson relation $\sigma = 1.1\ \mathrm{km\ s^{-1}} \left(\frac{L}{1\ \mathrm{pc}}\right)^{\eta_{\mathrm{turb}}}$, $\eta_{\mathrm{turb}} = 0.38$
- Turbulent cascade
  - Cascading power $\epsilon \approx \frac{\sigma^2}{\tau} \approx \frac{\sigma^3}{L} \approx \mathrm{constant}$
  - $\sigma \propto \epsilon^{1/3} L^{1/3}$, $\eta_{\mathrm{turb}} = 0.33$
- Energy equilibrium $\frac{GM^2}{L} \approx M\sigma^2$, $\sigma \propto M^{0.5} L^{-0.5} \propto L^{0.45}$, $\eta_{\mathrm{turb}} = 0.45$

Star formation in stellar clusters

More than 70% of stars form in groups (clusters)

- How to form stars with different masses?
- Where in the cluster do stars of different masses from?
- Is the mass segregation primordial?
- In-situ or conveyor belt formation of clusters?

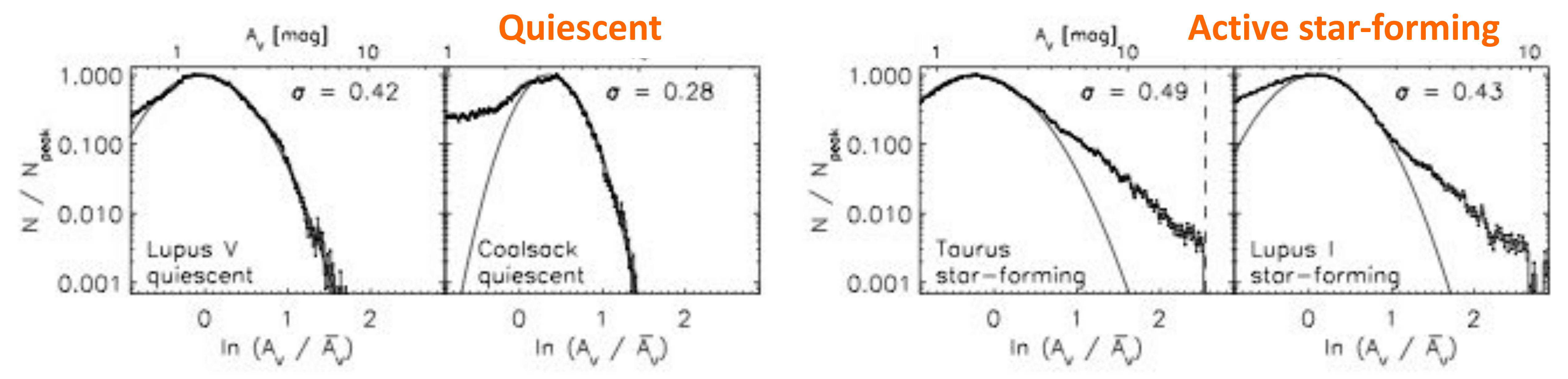


Rivilla 2013

Instead of pure Jeans fragmentation, the star-forming gas is structured!

Star formation in stellar clusters

Density Probability Density Function (PDF)



Kainulainen+ 2010

$$\mathscr{P}(\delta) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(\delta - \bar{\delta})^2}{2\sigma^2}\right),\ \delta = \log\left(\frac{\rho}{\bar{\rho}}\right)$$

Lognormal: PDF widening resulting from successive shocks

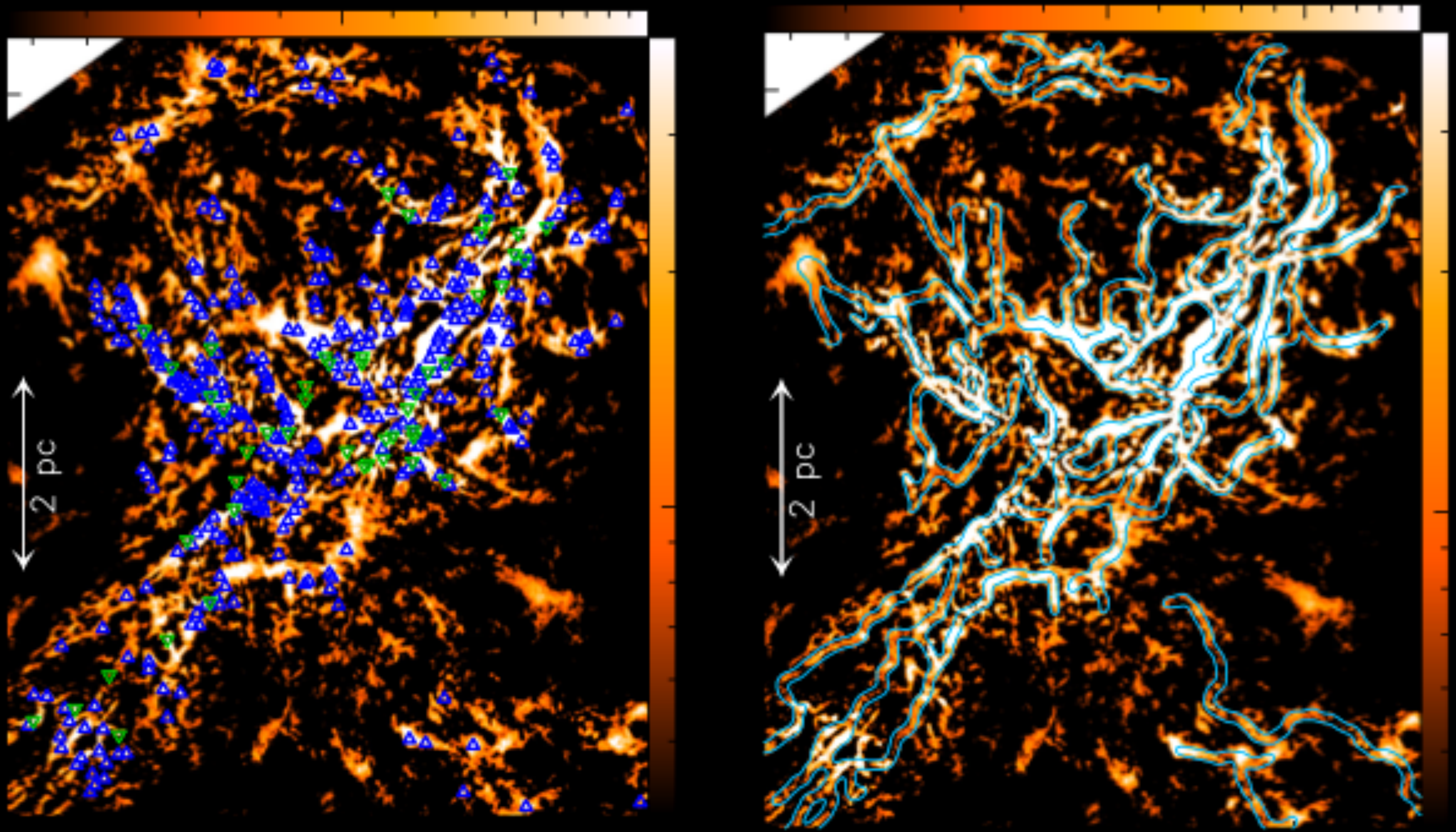
$$\mathscr{P}(\rho) = \mathscr{P}_0 \left(\frac{\rho}{\rho_0}\right)^{-p}$$

Powerlaw: from a powerlaw density profile

Star formation in stellar clusters

The molecular cloud structure and star formation

Prestellar cores (stars) are highly correlated with filamentary structures



Konyves+ 2015 (Polaris star-forming region)

Star formation in stellar clusters

A proof of dimensional change

- The fractal dimension is a space-filling factor $M \propto L^D$
- The interstellar medium (ISM) is fractal $D \sim 2 - 3$

  (Elmegreen & Falgarone 1996; Elmegreen 1997, 2002; Sánchez et al. 2005; Sánchez & Alfaro 2008; Kainulainen et al. 2011; Gusev 2014; Lee et al. 2016; Federrath et al. 2009; Konstandin et al. 2016; Padoan et al. 2016; Fleck 1996)

Clouds -> Sheets -> Filaments -> Cores

We knew that stars form in filaments, even far before the filaments are observationally demonstrated!

Star formation in stellar clusters

Mass function of astrophysical objects

- What is a mass function? (2 common equivalent forms)
  - $\frac{dN}{dM} \propto M^{-\alpha}$
  - $\frac{dN}{d\log M} \propto M^{\Gamma}$

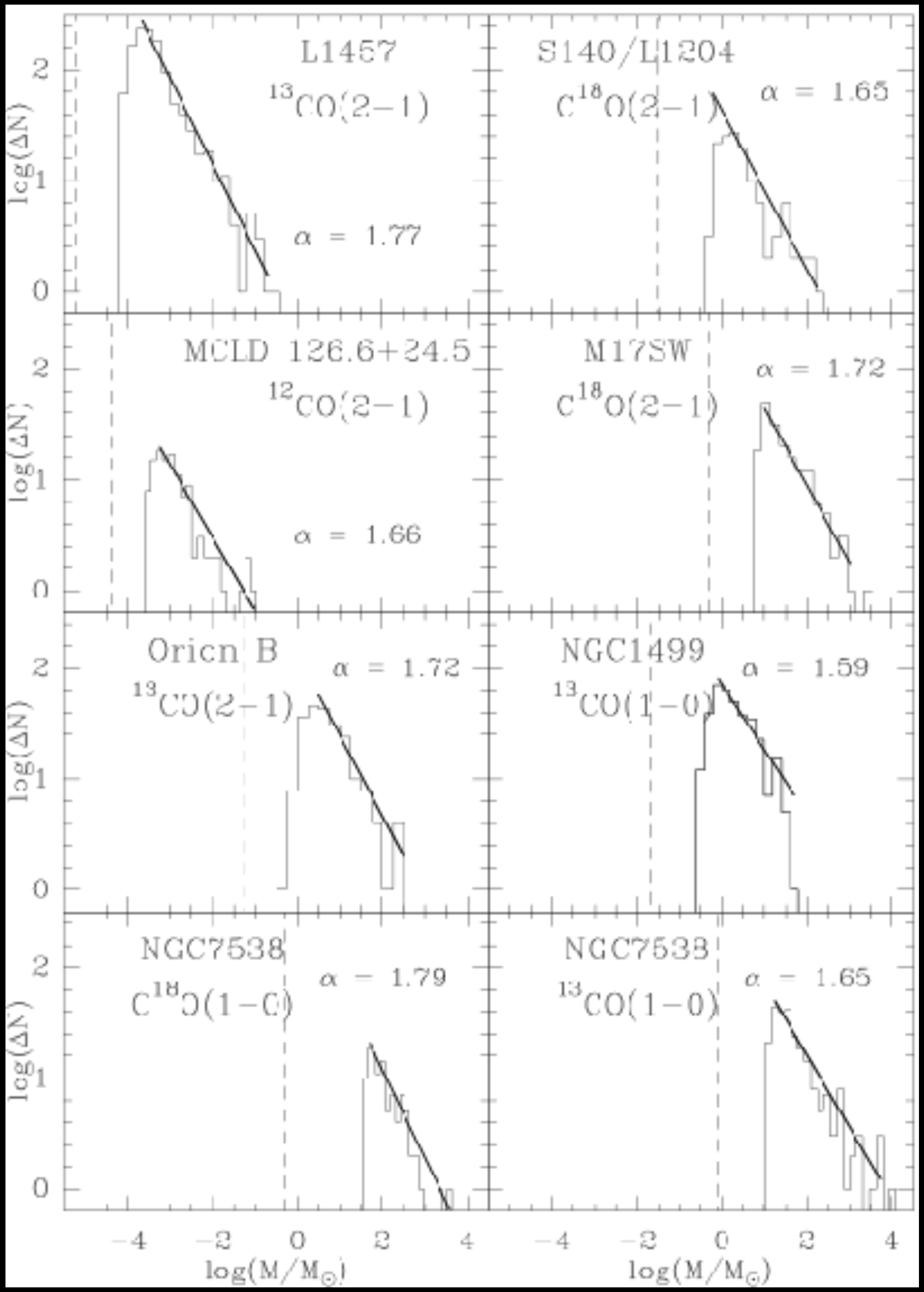
- If we consider wavenumber $k = 2\pi/L$
  - $M \propto \rho L^3 \propto \rho k^{-3} \approx \frac{M_{\text{total}}}{N(M)}$
  - $\frac{dN(M)}{dM} \propto \frac{k^3}{\rho M} \propto M^{-2}$

Star formation in stellar clusters

Mass function of CO clumps

Molecular clouds (H_2) are observed with CO line emissions

$$\alpha \approx 1.6$$



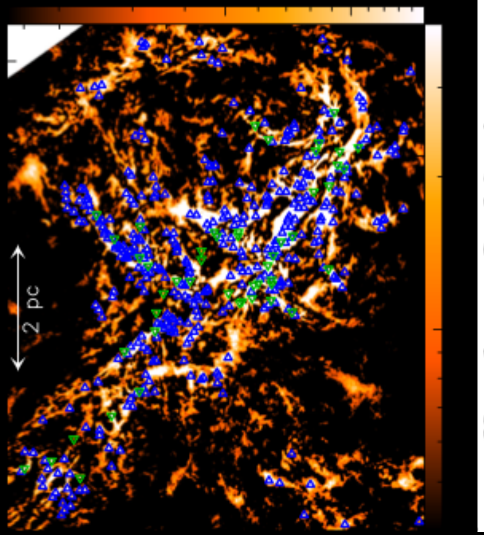
Kramer 1998

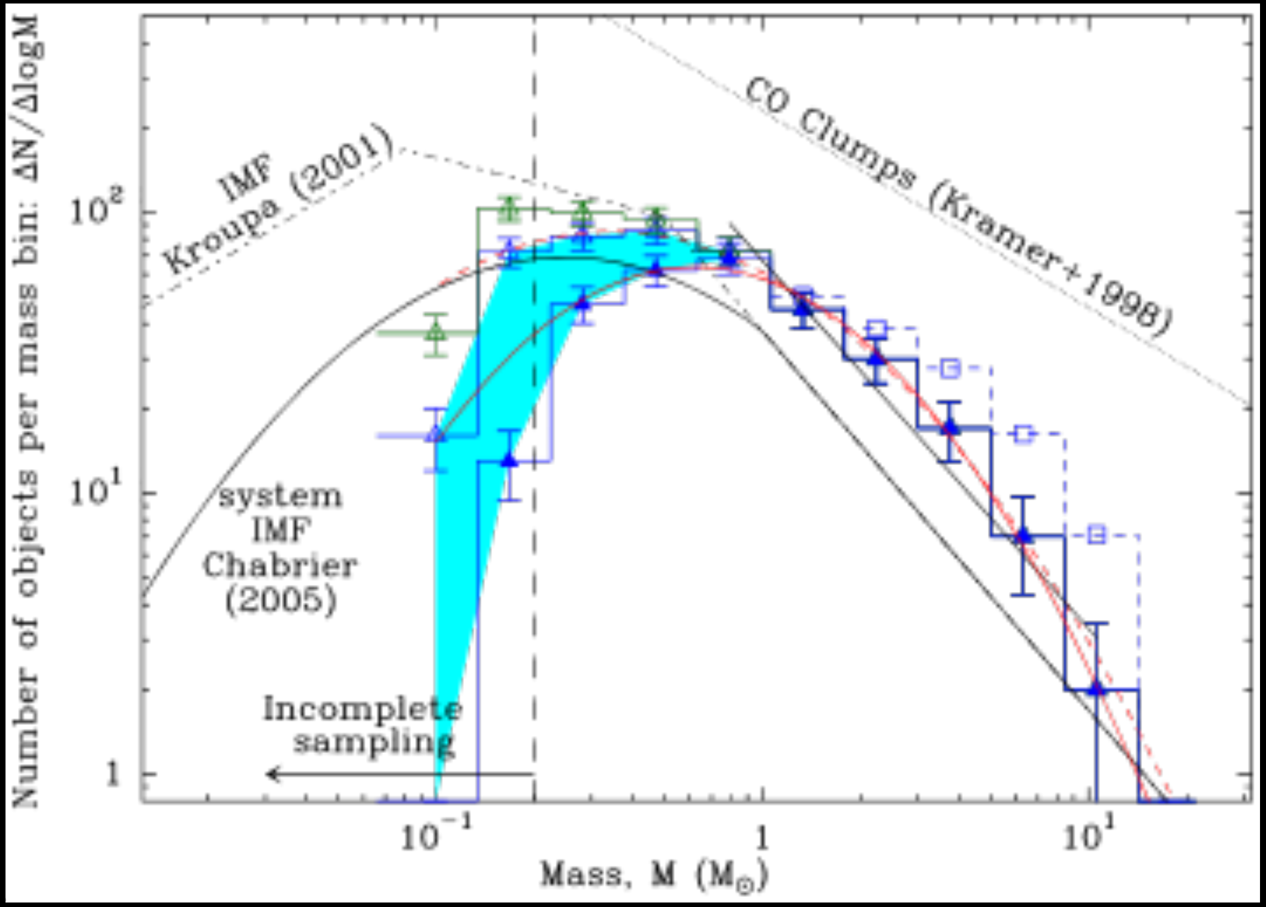
Star formation in stellar clusters

Mass function of prestellar cores

- Prestellar cores are dense small objects that are star-forming
- Fragmentation could happen during the collapse

$$\alpha \approx 2.31$$





Konyves+ 2015

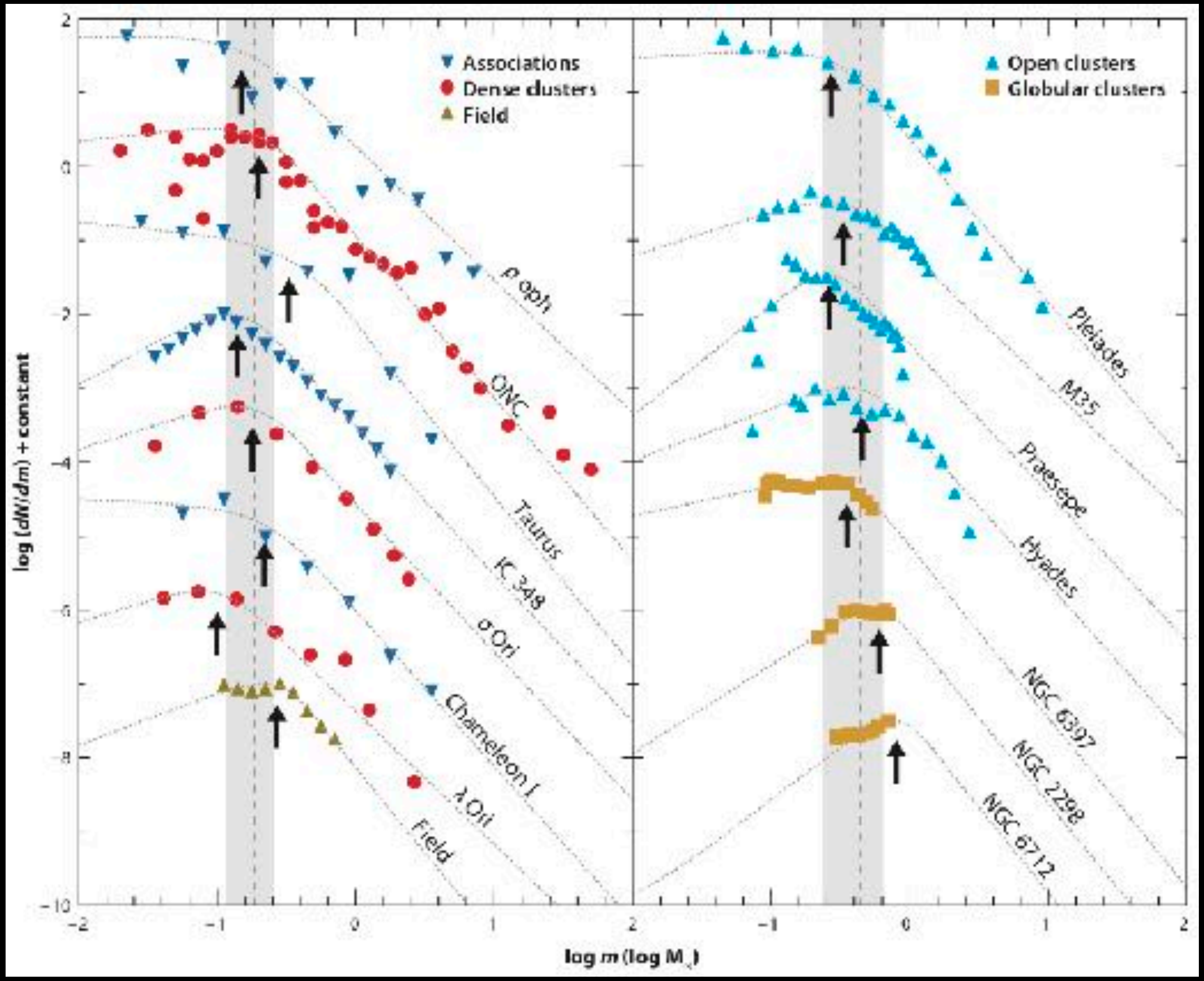
Star formation in stellar clusters

Mass function of new-born stars: the Initial Mass Function (IMF)

- The Initial Mass Function (IMF) describes the stellar mass distribution at their birth

- Universality seems to hold in a wide variety of star-forming environments (Salpeter 1955, Kroupa 2002, Chabrier 2003, Hillenbrand 2004, Moraux+2007, Bastien+2010, Offner+2014)
  - Peak at $0.2\ M_\odot$
  - $dN/dM \propto M^{-2.35}$
  - $dN/d\log M \propto M^{-1.35}$

$$\alpha \approx 2.35$$



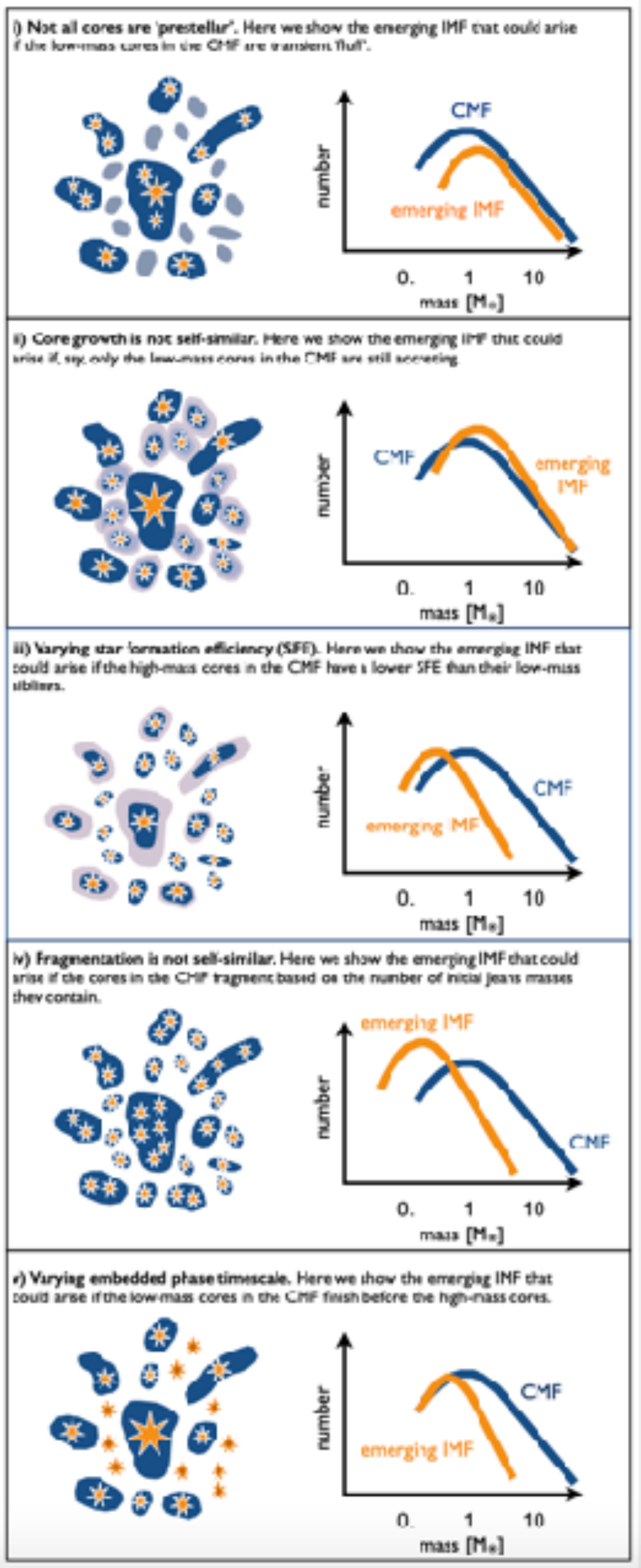
Bastien+ 2010

Star formation in stellar clusters

From the CMF to the IMF

- Very similar in shape
- Difference in peak mass (3x)
- Slight difference in slope (2.31 vs 2.35)

- Possible reasons
  - Fecundity of cores
  - Core growth
  - Core star formation efficiency (SFE)
  - Core fragmentation ($M_\mathrm{J} \propto \rho^{-1/2}$)
  - Collapse timescale ($t_\mathrm{ff} \propto \rho^{-1/2}$)



i) Not all cores are 'prestellar'. Here we show the emerging IMF that could arise if the low-mass cores in the CMF are transient 'fluff'.

ii) Core growth is not self-similar. Here we show the emerging IMF that could arise if, say, only the low-mass cores in the CMF are still accreting.

iii) Varying star formation efficiency (SFE). Here we show the emerging IMF that could arise if the high-mass cores in the CMF have a lower SFE than their low-mass siblings.

iv) Fragmentation is not self-similar. Here we show the emerging IMF that could arise if the cores in the CMF fragment based on the number of initial Jeans masses they contain.

v) Varying embedded phase timescale. Here we show the emerging IMF that could arise if the low-mass cores in the CMF finish before the high-mass cores.

Star formation in stellar clusters

Why do we care about the IMF?

- High mass stars dominate in luminosity $L \propto M^{2-4}$
- Low mass stars host planets
- High mass stars feedback energy to the environment
  - Ionizing photons
  - Winds, outflows
  - Protostellar jets
  - Supernovae -> metals
- Sub-grid model for large-scale simulations
- Conversion from observed luminosity to stellar mass

$$L_{\rm total} = \int L(M) \frac{dN}{dM} dM, \quad M_{\rm total} = \int M \frac{dN}{dM} dM$$

Major pending puzzles in star formation

Selected somehow following personal tastes

- Origin of Globular Clusters (GC)/Young Massive Clusters (YMC)
- Is the Initial Mass Function really universal?
- How and where do massive stars form?
- When does mass segregation happen in clusters?
- Are massive stars a scaled-up version of lower-mass stars?
- How does feedback regulate star formation at stellar scales, cluster scales, and galactic scales?
- How does the protoplanetary disk co-evolve with the host star?
- The role of interstellar dust during star formation