



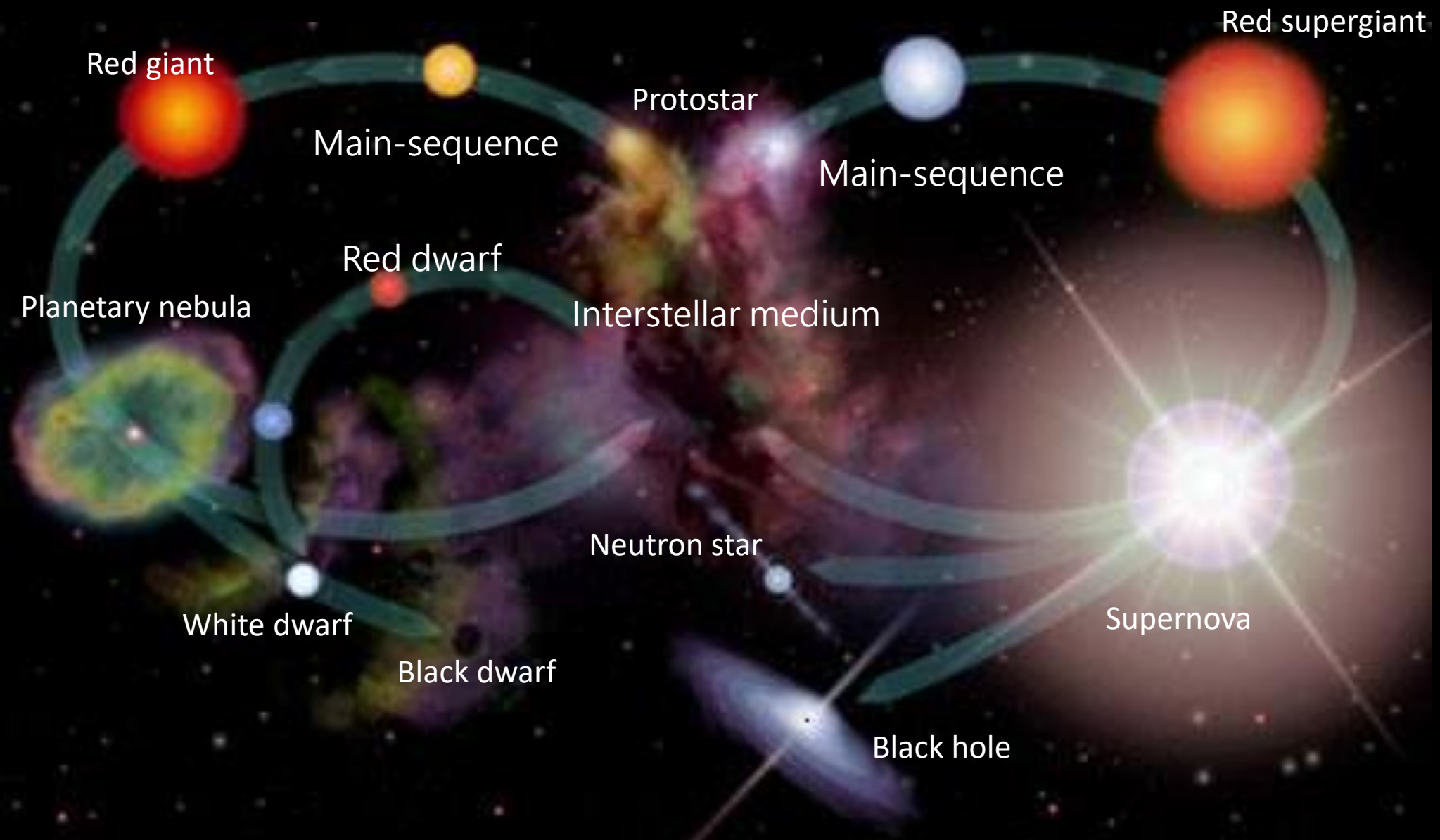
Compact Objects: Neutron Stars and Black Holes

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NCTS Summer School, July 04, 2023

Low-mass stars

High-mass stars



Black Hole – Schwarzschild Radius

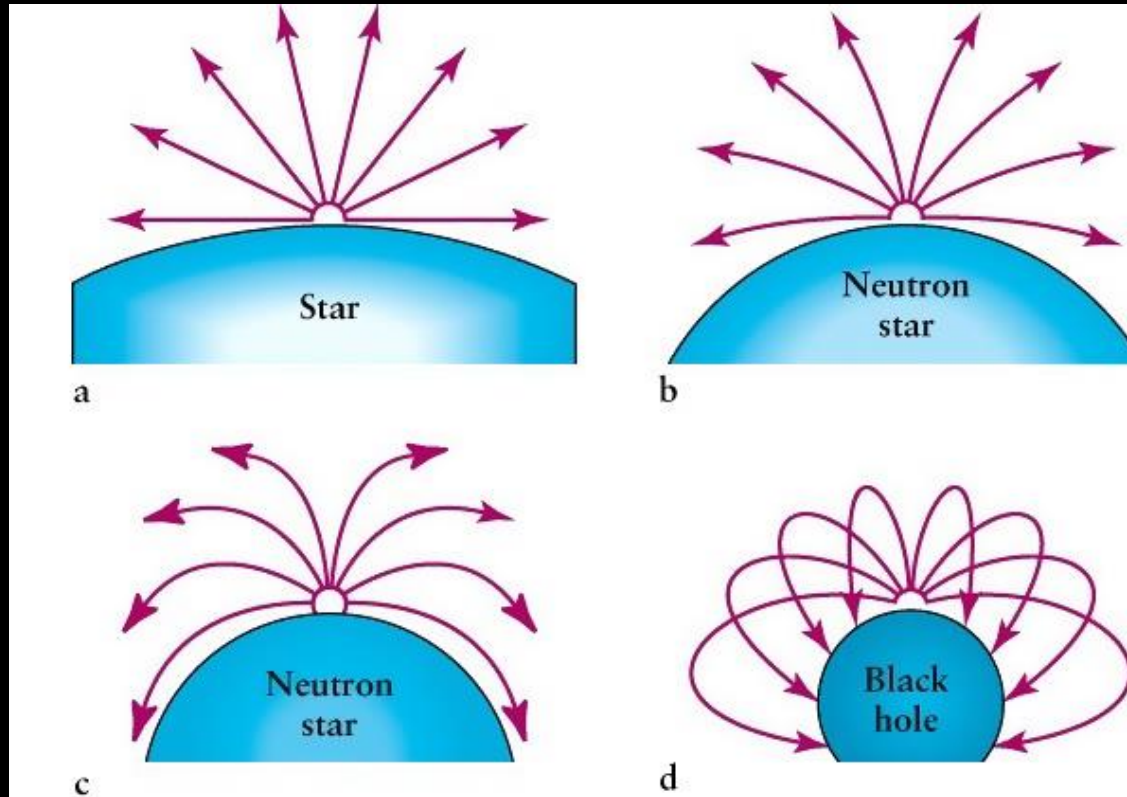
- In 1915, A. Einstein published General Relativity
 - In 1916, K. Schwarzschild derive a first non-trivial solution
 - Any object can be a black hole as long as its radius is smaller than the Schwarzschild radius

$$R_{Sch} = \frac{2GM}{c^2}$$

Object	Size	mass (kg)	R_{Sch}	density (g/cm ³)
Earth	6400 km	6×10^{24}	0.9cm	2×10^{27}
Sun	700,000 km	2×10^{30}	3 km	1.8×10^{16}
Milky Way	~20 kpc	$\sim 2 \times 10^{42}$	20,000 AU	1.8×10^{-8}
Observable Universe	$\sim 3 \times 10^9$ pc	$\sim 10^{53}$	5×10^9 pc	7.3×10^{-30}

Black Hole – Event Horizon

- The escape velocity is equal to the speed of light
- A singularity (?) inside



Beyond White Dwarf

- In 1930. S. Chandrasekhar calculate the solution for degenerate Fermi gas, and derive the mass upper limit of a white dwarf
 - But A. Eddington does not believe (the existence of black hole)

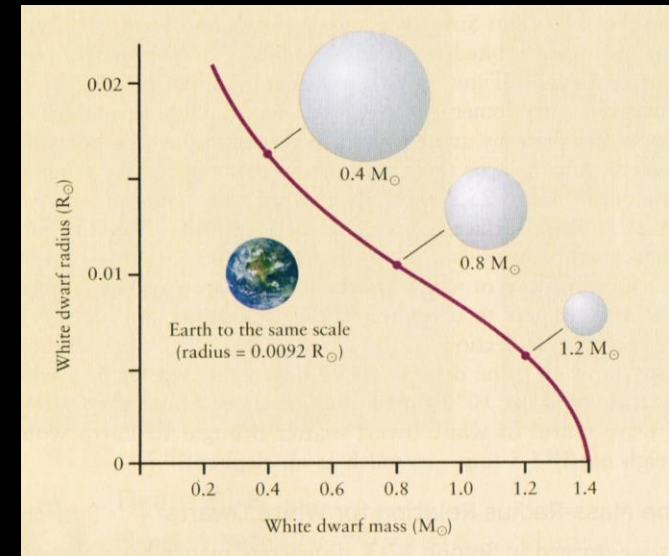
A white dwarf, i.e., the star is supported by the degeneracy pressure of the electrons, could be described by a polytrope of index $n = 1.5$ ($\gamma = 5/3$) and $K = K_1$.

From the mass-radius relation

$$\left(\frac{GM}{M_n}\right)^{n-1} \left(\frac{R}{R_n}\right)^{3-n} = \frac{[(n+1)K]^n}{4\pi G}$$

We know that

$$R \propto M^{-1/3}$$



The Chandrasekhar Mass

Therefore, the mean density increases as the square of the mass

$$\bar{\rho} \propto MR^{-3} \propto M^2$$

When M increases, R decreases and $\bar{\rho}$ increases.

Eventually, the density will become so high that the electron gas will turn to be **relativistic**.

The equation of state become a polytrope with $n = 3$ (or $\gamma = 4/3$) with $K = K_2$.

The Chandrasekhar Mass

For $n = 3$, there is only one solution determined by K

$$M = M_{Ch} = 4\pi M_3 \left(\frac{K_2}{\pi G} \right)^{\frac{3}{2}} = \frac{M_3 \sqrt{1.5}}{4\pi} \left(\frac{hc}{G m_H^{4/3}} \right)^{3/2} \mu_e^{-2}$$

Where μ_e^{-1} is the number of free electrons per nucleon

$$\frac{1}{\mu_e} \equiv \sum_i X_i \frac{Z_i}{A_i} \approx \frac{1}{2} (1 + X)$$

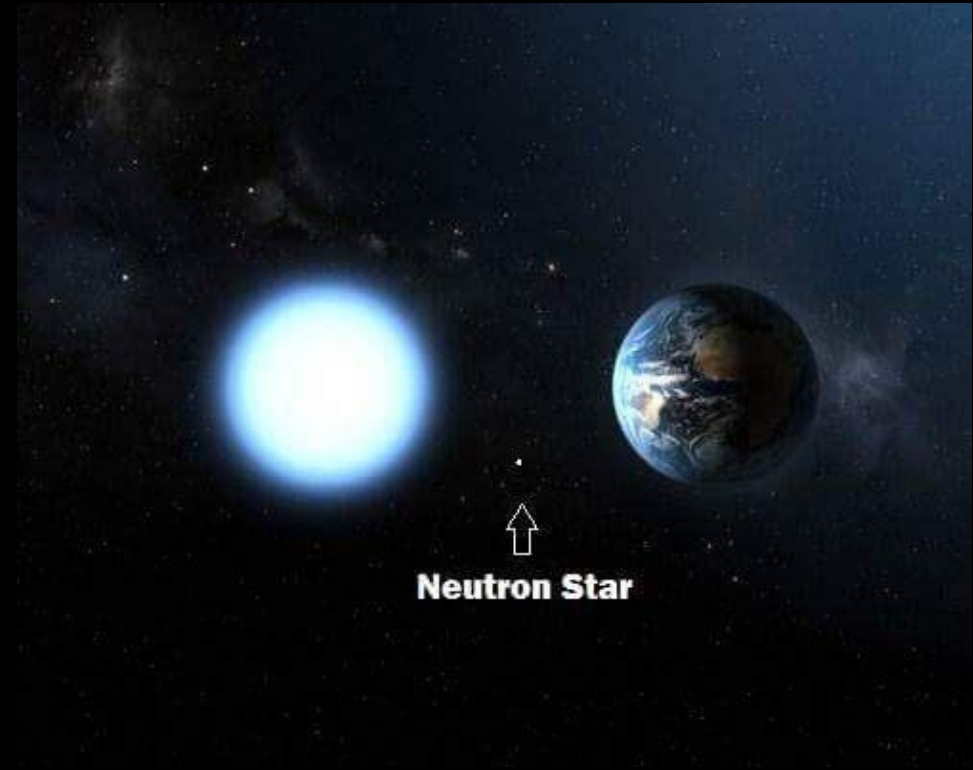
X_i : mass fraction of a species, Z_i : number of charge of a particle,
 A_i : atomic mass

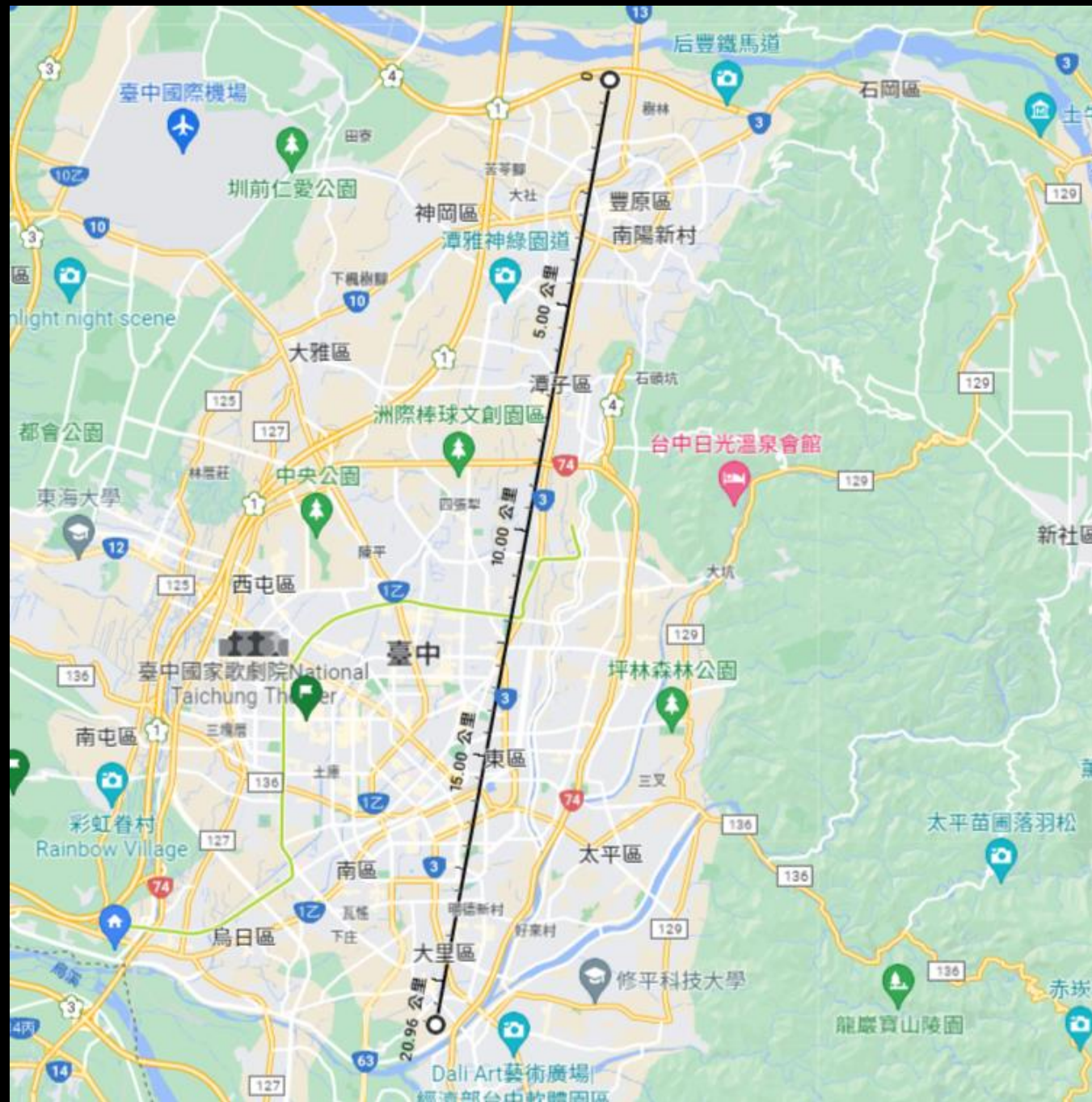
Beyond White Dwarf

- In 1932, J. Chadwick found neutron
- In 1934, W. Baade and F. Zwicky proposed the concept of neutron stars.
- In 1939, J. R. Oppenheimer and G. M. Volkoff derived an equation of state, and estimated the size of a neutron star.
 - But astronomers do not believe that (the thermal emission of the) neutron star is detectable.
 - The temperature of neutron star is very high, and the emission is mainly in the X-ray band

Neutron Stars

- A neutron star forms when the electron's degeneracy pressure cannot resist the collapse
 - $M_{NS} \gtrsim 1.4M_{\odot}$
 - Chandrasekhar limit
 - Though recent studies found a few "light weight" neutron stars
 - Degeneracy pressure of neutrons





Neutron Star: Discovery

- In 1967, J. B. Burnell and A. Hewish detected pulsed radio signals from the sky
 - 1960s is the golden era of radio astronomy
 - Pulsar (**pulsating** radio sources)
 - The emission is nonthermal



Observation of a Rapidly Pulsating Radio Source

by

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S. J. BELL
J. D. H. PILKINGTON
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Mullard Radio Astronomy Observatory,
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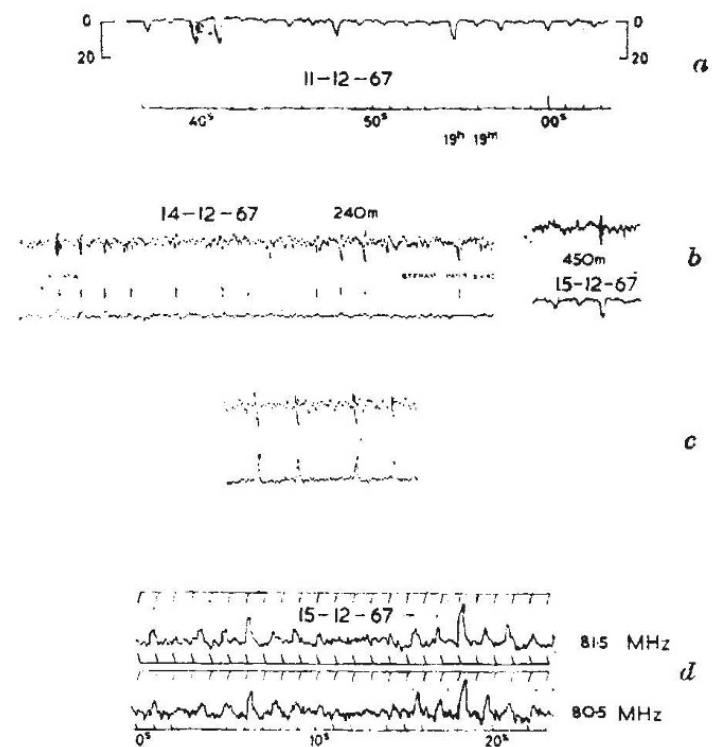
Unusual signals from pulsating radio sources have been recorded at the Mullard Radio Astronomy Observatory. The radiation seems to come from local objects within the galaxy, and may be associated with oscillations of white dwarf or neutron stars.

In July 1967, a large radio telescope operating at a frequency of 81.5 MHz was brought into use at the Mullard Radio Astronomy Observatory. This instrument was designed to investigate the angular structure of compact radio sources by observing the scintillation caused by the irregular structure of the interplanetary medium¹. The initial survey includes the whole sky in the declination range $-08^\circ < \delta < 44^\circ$ and this area is scanned once a week. A large fraction of the sky is thus under regular surveillance. Soon after the instrument was brought into operation it was noticed that signals which appeared at first to be weak sporadic interference were repeatedly observed at a fixed declination and right ascension; this result showed that the source could not be terrestrial in origin.

Systematic investigations were started in November and high speed records showed that the signals, when present, consisted of a series of pulses each lasting ~ 0.3 s and with a repetition period of about 1.337 s which was soon found to be maintained with extreme accuracy. Further observations have shown that the true period is constant to better than 1 part in 10^7 although there is a systematic variation which can be ascribed to the orbital motion of the Earth. The impulsive nature of the recorded signals is caused by the periodic passage of a signal of descending frequency through the 1 MHz pass band of the receiver.

The remarkable nature of these signals at first suggested an origin in terms of man-made transmissions which might arise from deep space probes, planetary radar or the reflexion of terrestrial signals from the Moon. None of

of these unusual sources in terms of the stable oscillations of white dwarf or neutron stars is proposed.



Why a neutron star rotating so fast?



Why a neutron star rotating so fast?

The moment of inertia of a rigid body (sphere) is

$$I = \frac{2}{5}MR^2$$

Conservation of Angular momentum:

$$L = I\omega = \text{constant}$$

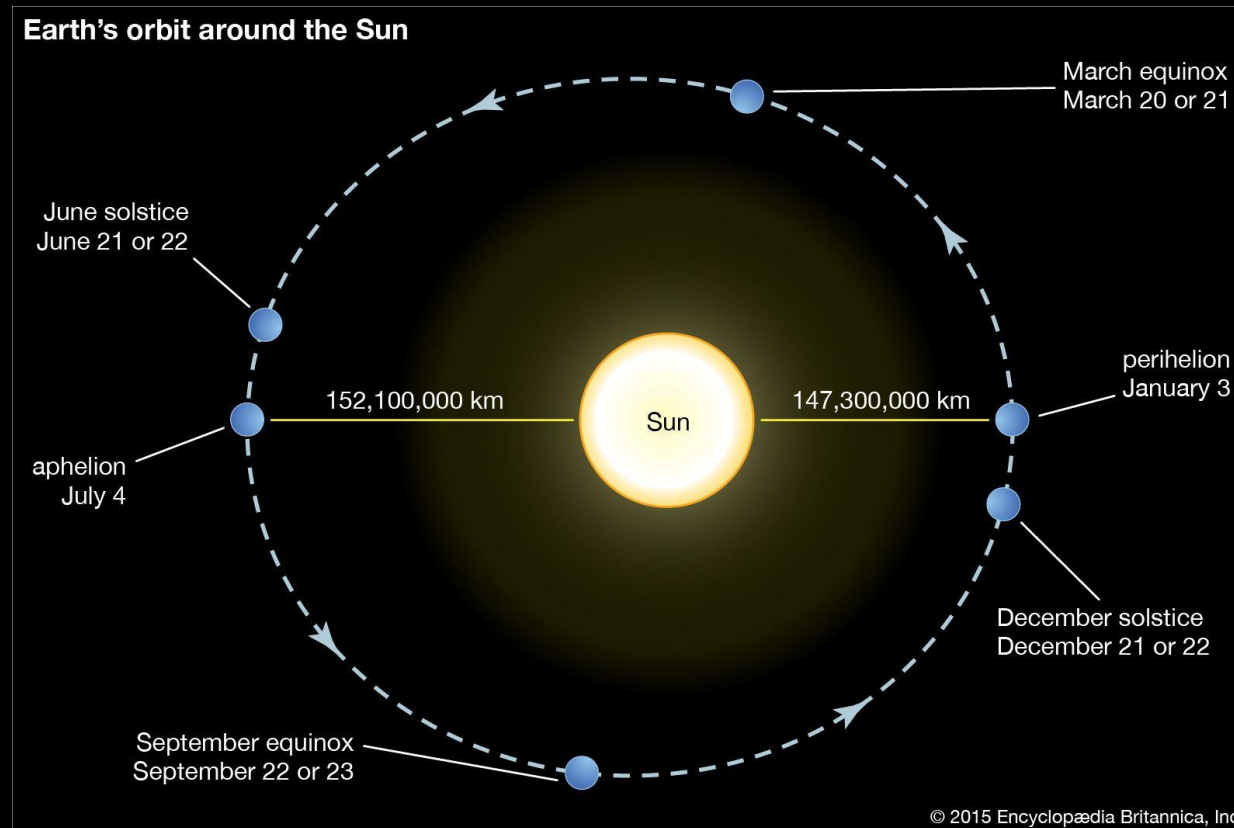
$$I_i\omega_i = I_f\omega_f$$

Q: If our Sun collapses to form a neutron star, estimate the rotating frequency (or period)

$$(M_{\odot} \sim 2 \times 10^{30} \text{ kg}, R_{\odot} \sim 700000 \text{ km}, P_{\odot} \sim 1 \text{ month})$$

Fast Rotating = Neutron Star?

- Imagine that: the moving velocity of the Earth increased a little bit...



- How fast can the Earth rotate?



Can neutron star rotating so fast?

The upper limit of the rotating speed is the Keplerian velocity:

$$4\pi^2 R \cdot v_{bk}^2 = \frac{GM}{R^2}$$

$$\Rightarrow v_{bk} = \sqrt{\frac{GM}{4\pi^2 R^3}} = 2170 \left(\frac{M}{1.4M_{\odot}} \right)^{\frac{1}{2}} \left(\frac{R}{10km} \right)^{-\frac{3}{2}} \text{ Hz}$$

$$\Rightarrow P_{bk} = 0.46 \left(\frac{M}{1.4M_{\odot}} \right)^{-\frac{1}{2}} \left(\frac{R}{10km} \right)^{\frac{3}{2}} \text{ ms}$$

Breaking Frequency (Period)

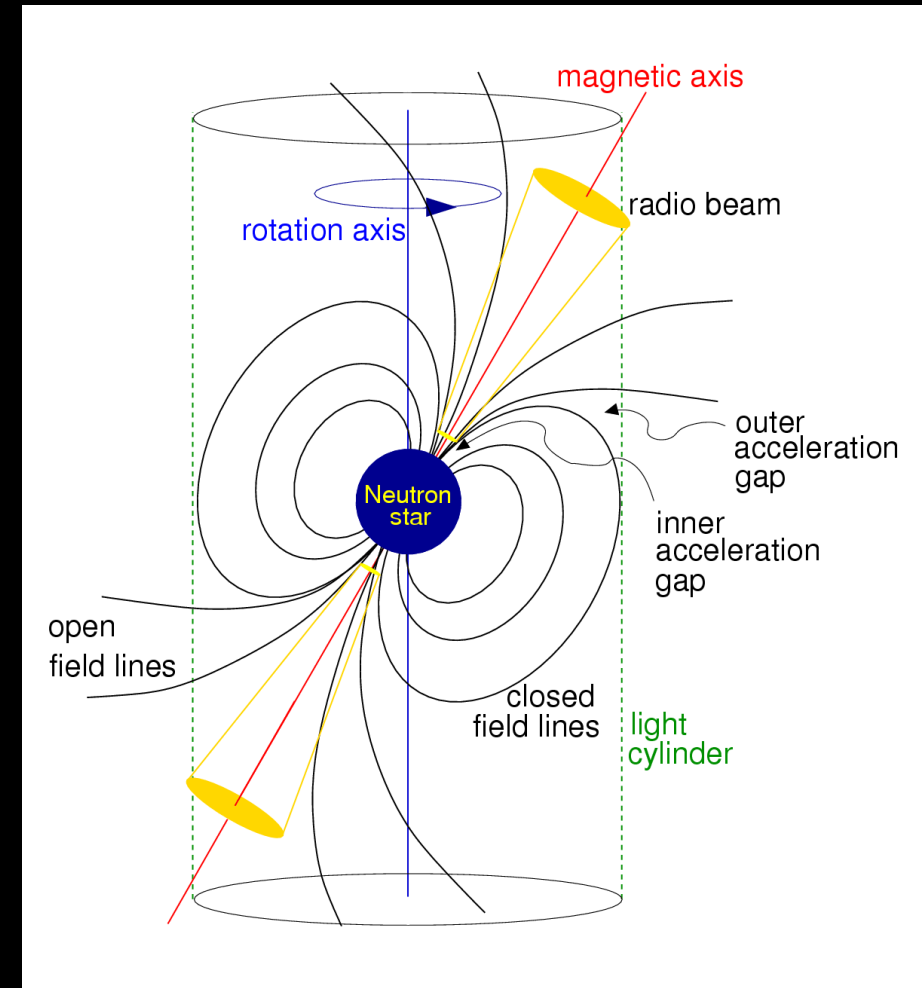
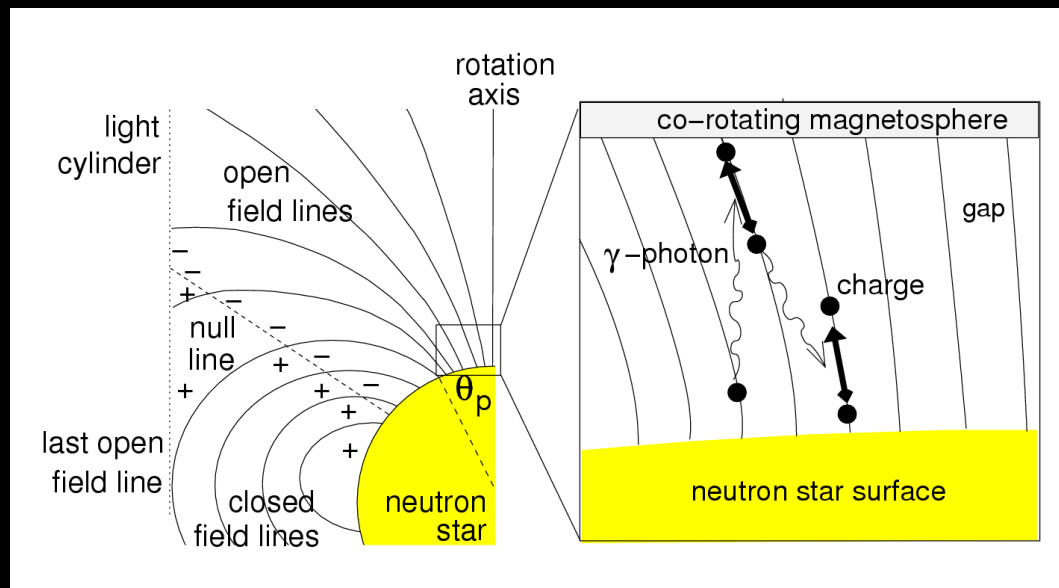
Objects	Mass	Radius	Breaking period
Sun	$1 M_{\odot}$	700,000 km	2.8 hours
White dwarf	$1 M_{\odot}$	6,000 km	8 s
Neutron star	$1.4 M_{\odot}$	10 km	0.46 ms

Where is the energy from?

- The core of neutron stars do not have nuclear reaction
- What we observed is the radio emission

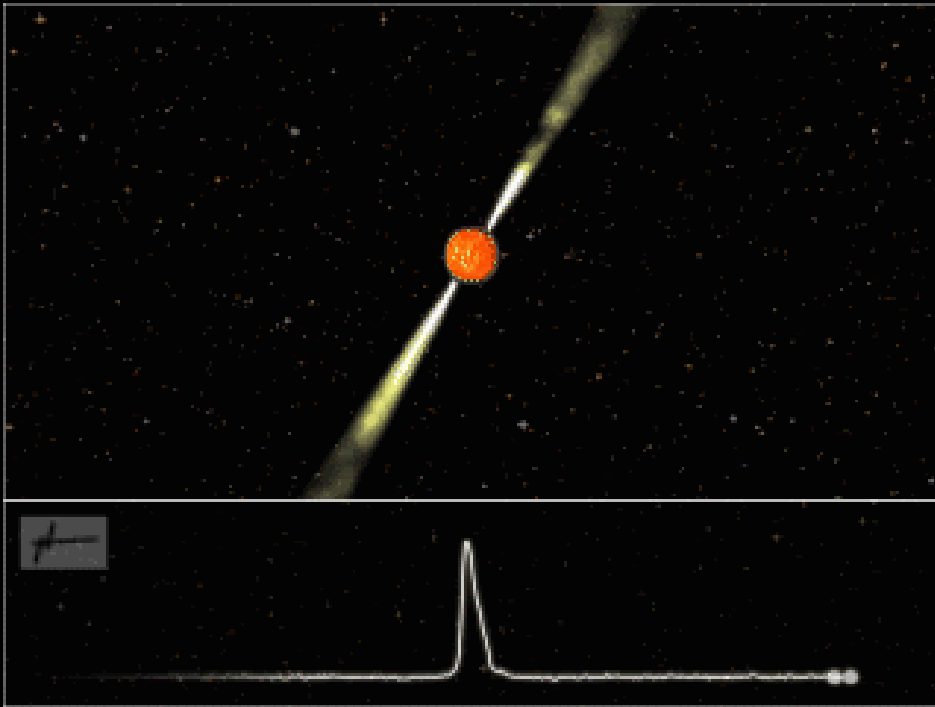
Energy Source

- Pacini 1967: strongly magnetized neutron stars could release their rotational energy and produce a large flow of relativistic particles

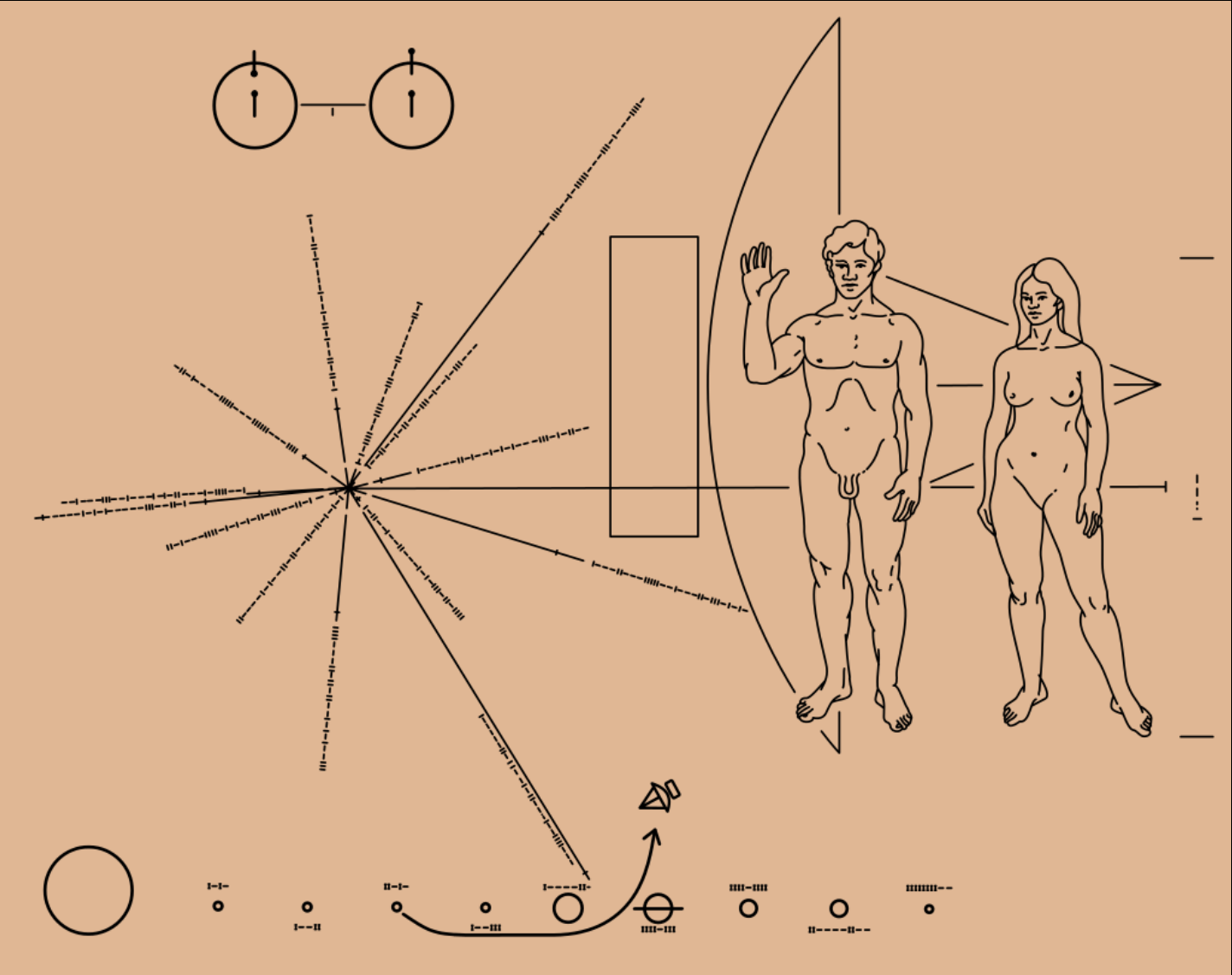


Pulsars

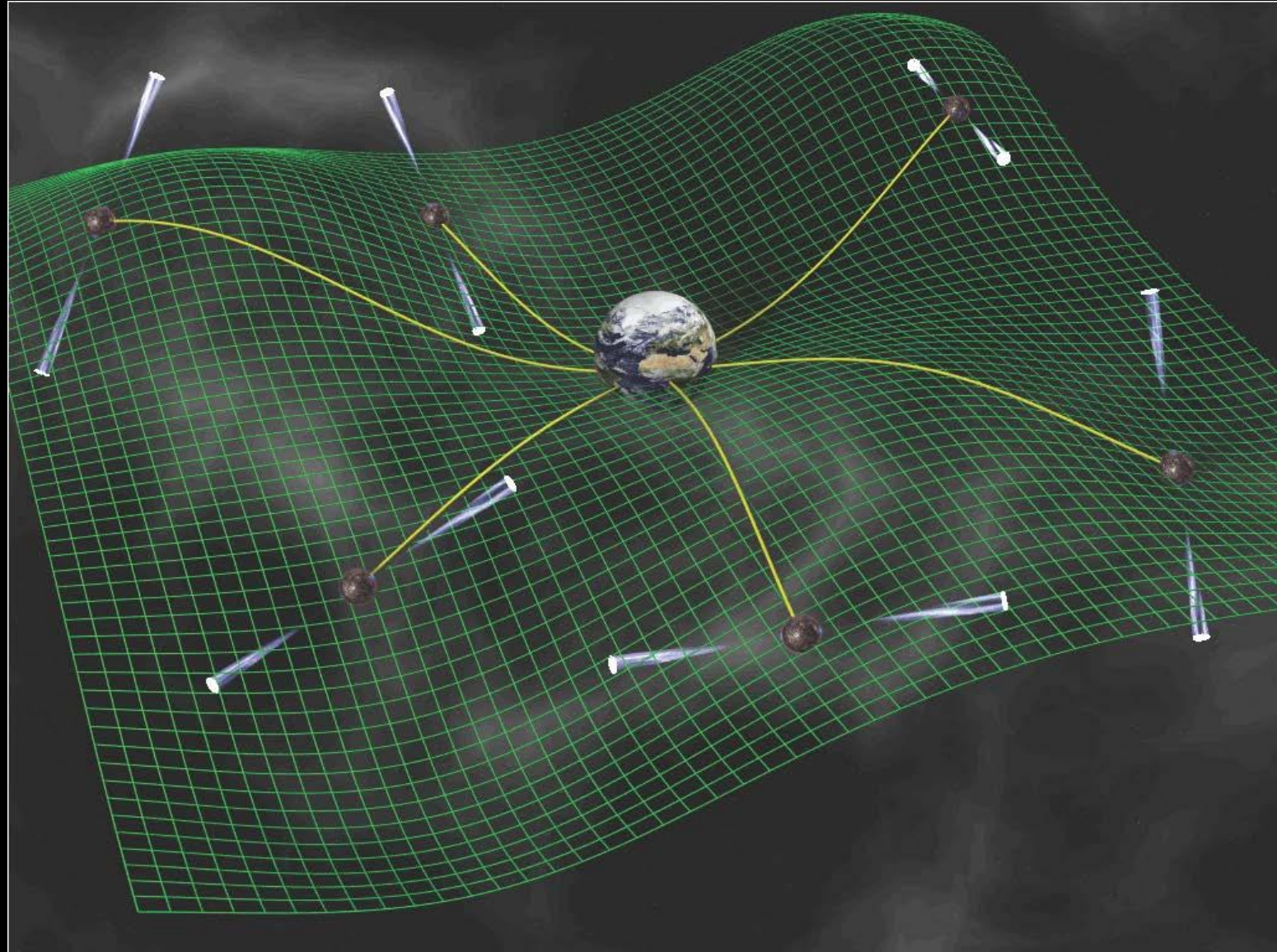
- Strongly magnetized, fast rotating neutron stars
 - Light house effect



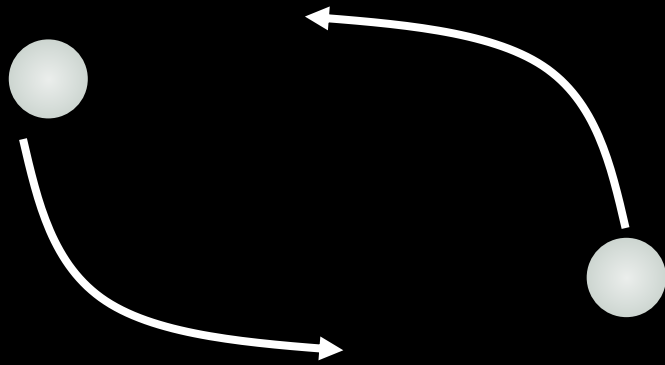
Pioneer Plaque



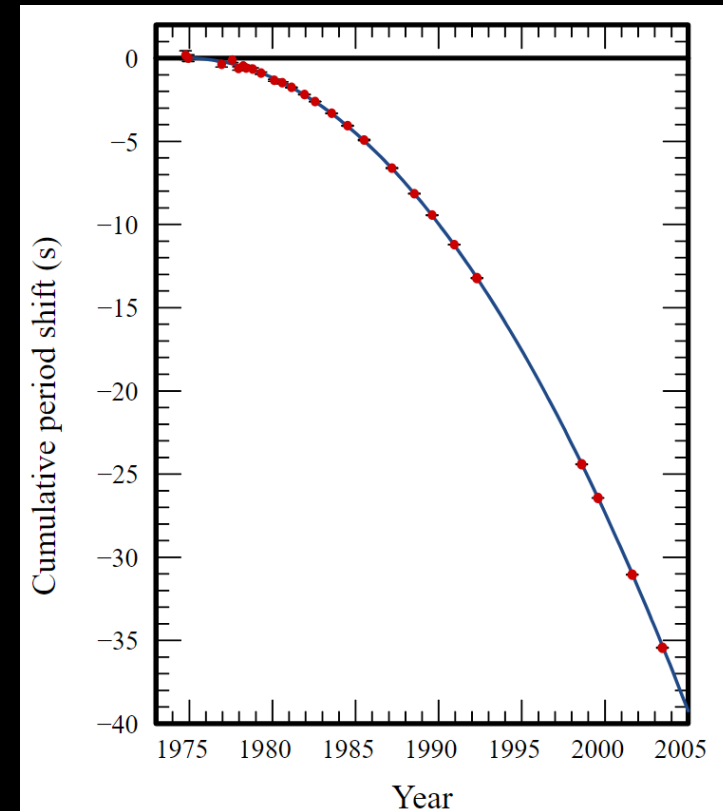
Pulsar Timing Array



Gravitational Wave from Binary Neutron Stars



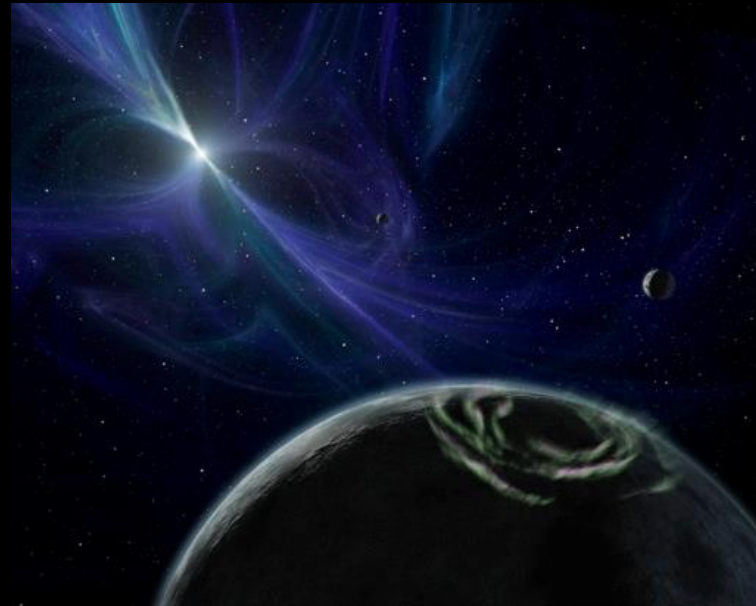
- Hulse-Taylor binary (PSR B1913+16)
 - $m_1 = 1.4M_{\odot}, m_2 = 1.36M_{\odot}$
 - $P_{orb} = 7.75$ hrs, $\epsilon = 0.617$
 - Prediction from general relativity: the orbit shrinks with the rate of 3 mm/orbit
 - 1993 Nobel Prize



Weisberg & Taylor (2004)

Search for Exoplanet

- Radial velocity: search for exoplanet by observing their effects on host stars
 - The spin period of pulsars have orbital Doppler effects!
 - PSR B1257+12 b ($0.02 M_{\oplus}$), c ($4.3 M_{\oplus}$), d ($3.9 M_{\oplus}$)



Energy Source

The energy source of an isolated neutron star is the rotational energy

$$E_{rot} = \frac{1}{2} I \omega^2 = \frac{2\pi^2 I}{P^2}$$

where I is the moment of inertia

$$I = \frac{2}{5} MR^2 \approx 10^{45} \text{ g cm}^2$$

Therefore, the energy lose rate is

$$\frac{dE_{rot}}{dt} = -\frac{4\pi^2 I \dot{P}}{P^3} = -L_{sd} \quad (\text{spin-down luminosity})$$

Magnetic Field Strength

Assume that the rotation energy releases in the form of magnetic dipole radiation:

$$\frac{4\pi^2 I \dot{P}}{P^3} = \frac{2}{3c} (BR^3 \sin \alpha)^2 \left(\frac{4\pi^2}{P^2} \right) \quad \text{Larmor's formula}$$

$$B^2 = \frac{3c^3 I P \dot{P}}{8\pi^2 R^6 \sin^2 \alpha}$$

Therefore, the lower limit of the magnetic field is

$$B > \left(\frac{3c^3 I P \dot{P}}{8\pi^2 R^6} \right)^{\frac{1}{2}} = 3.2 \times 10^{19} \left(\frac{P \dot{P}}{s} \right)^{\frac{1}{2}} (G) \quad \text{Spin-down inferred B field}$$

Characteristic Age

Assume that the neutron star rotates very fast in the beginning ($P_{t=0} = 0$), and B field changes slowly:

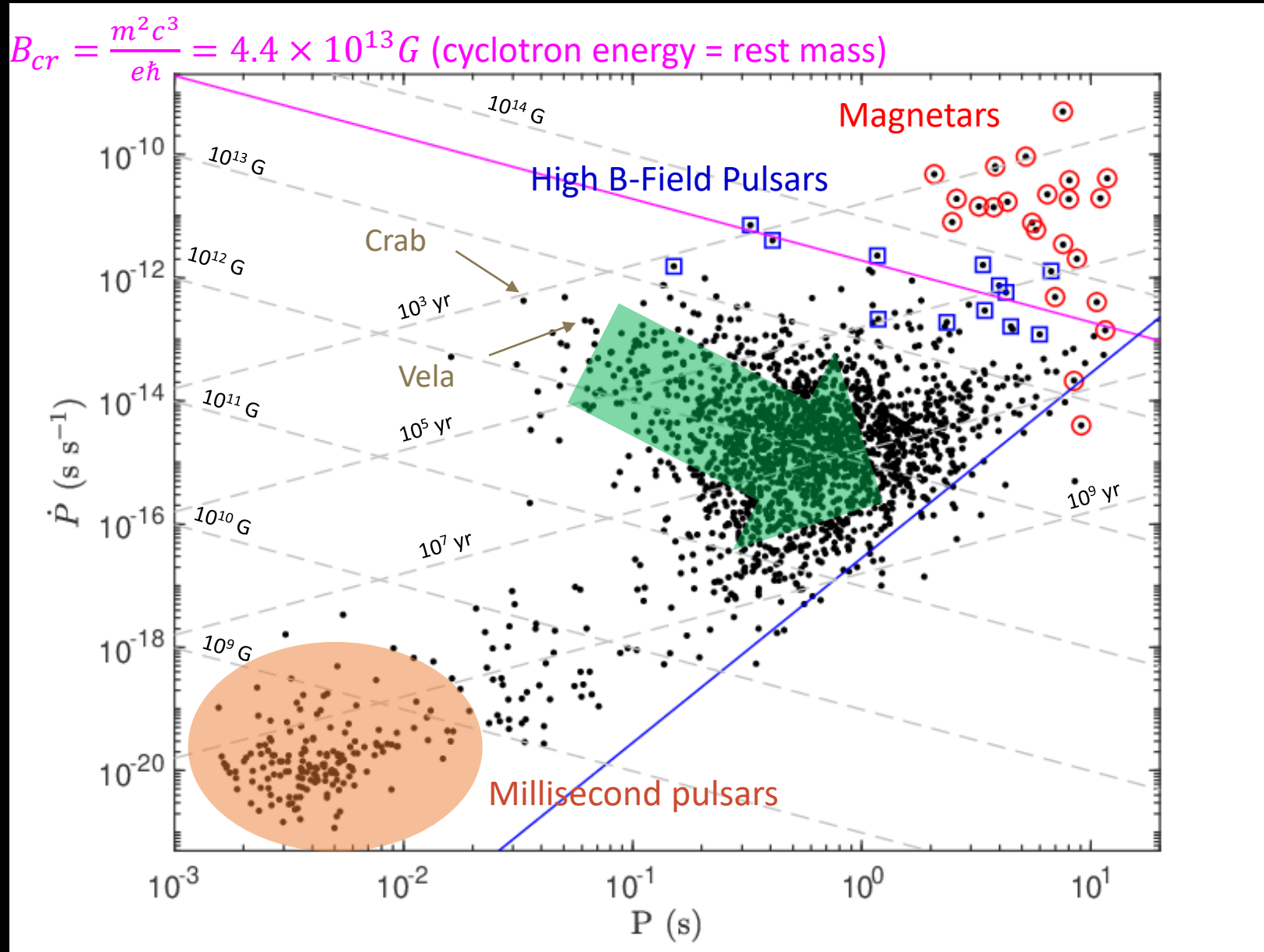
$$P\dot{P} = \frac{8\pi^2 R^6 (B \sin \alpha)^2}{3c^3 I} \quad \text{is a const}$$

Integrate this equation

$$\int_{P_0}^P P dP = \int_0^\tau P\dot{P} dt = P\dot{P} \int_0^\tau dt$$

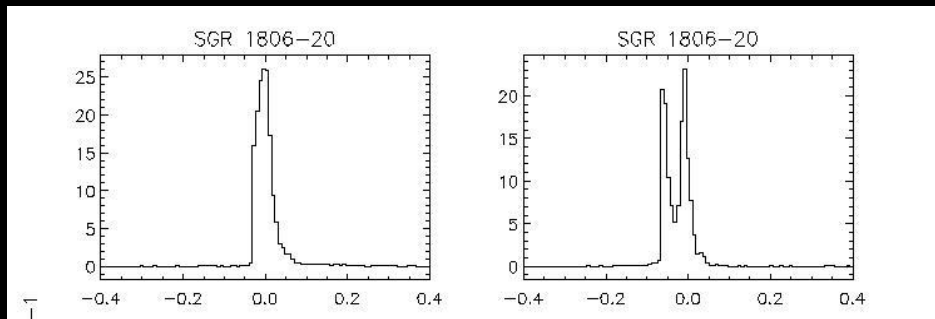
$$\Rightarrow \frac{P^2 - P_0^2}{2} = P\dot{P}\tau \Rightarrow \tau = \frac{P}{2\dot{P}}$$

P-Pdot Diagram

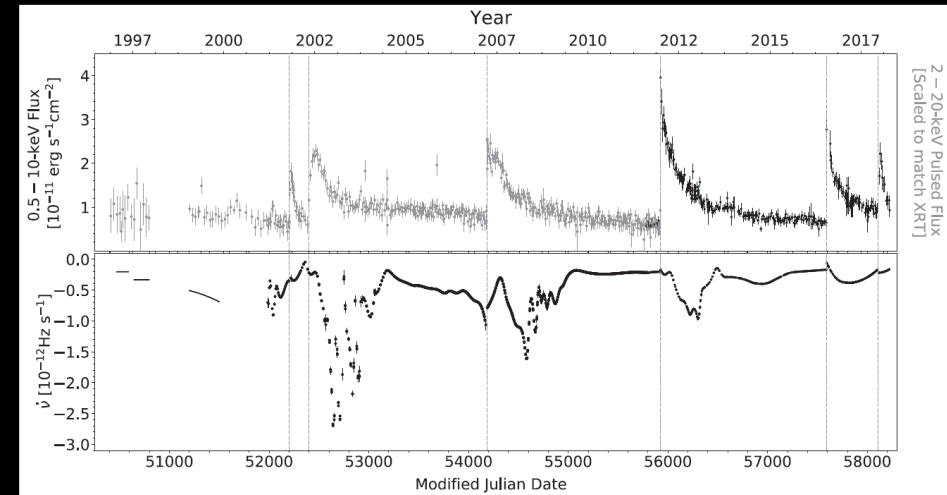


Magnetars

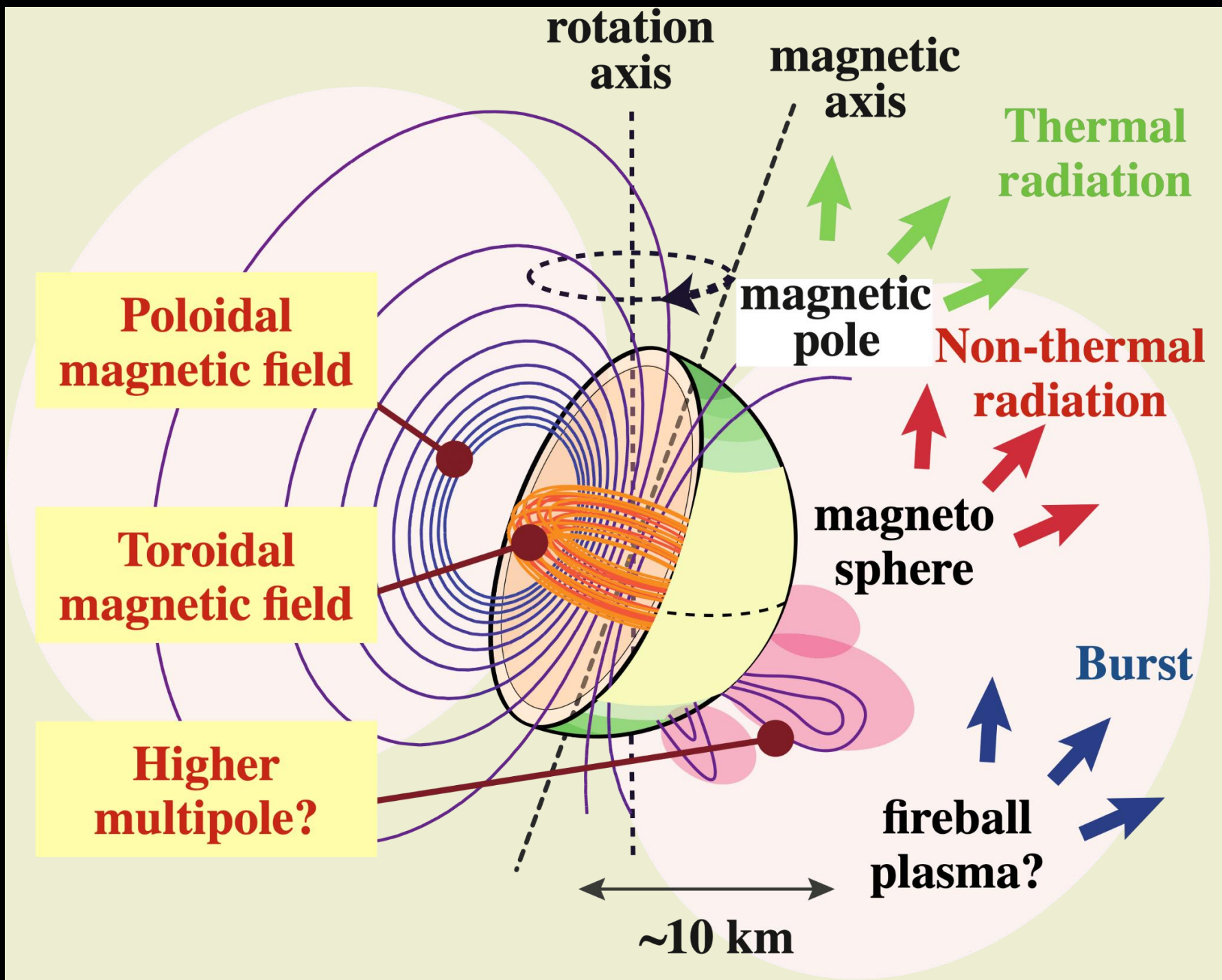
- After the launch of X-ray and Gamma-ray telescopes, astronomers found two special types of neutron stars
 - **Soft Gamma-ray Repeater**: exhibit short gamma-ray burst
 - **Anomalous X-ray Pulsar**: have strong X-ray variability, $L_X > L_{sd}$
- They were suggested to share the same origin
 - Strong magnetic fields (usually $B > 10^{14}$ G)
 - Although we found low-B-field magnetars and high-B-field rotation-powered pulsars
 - Long spin period (2-12 s)
 - High surface temperature
 - Energy source: the decay of B field



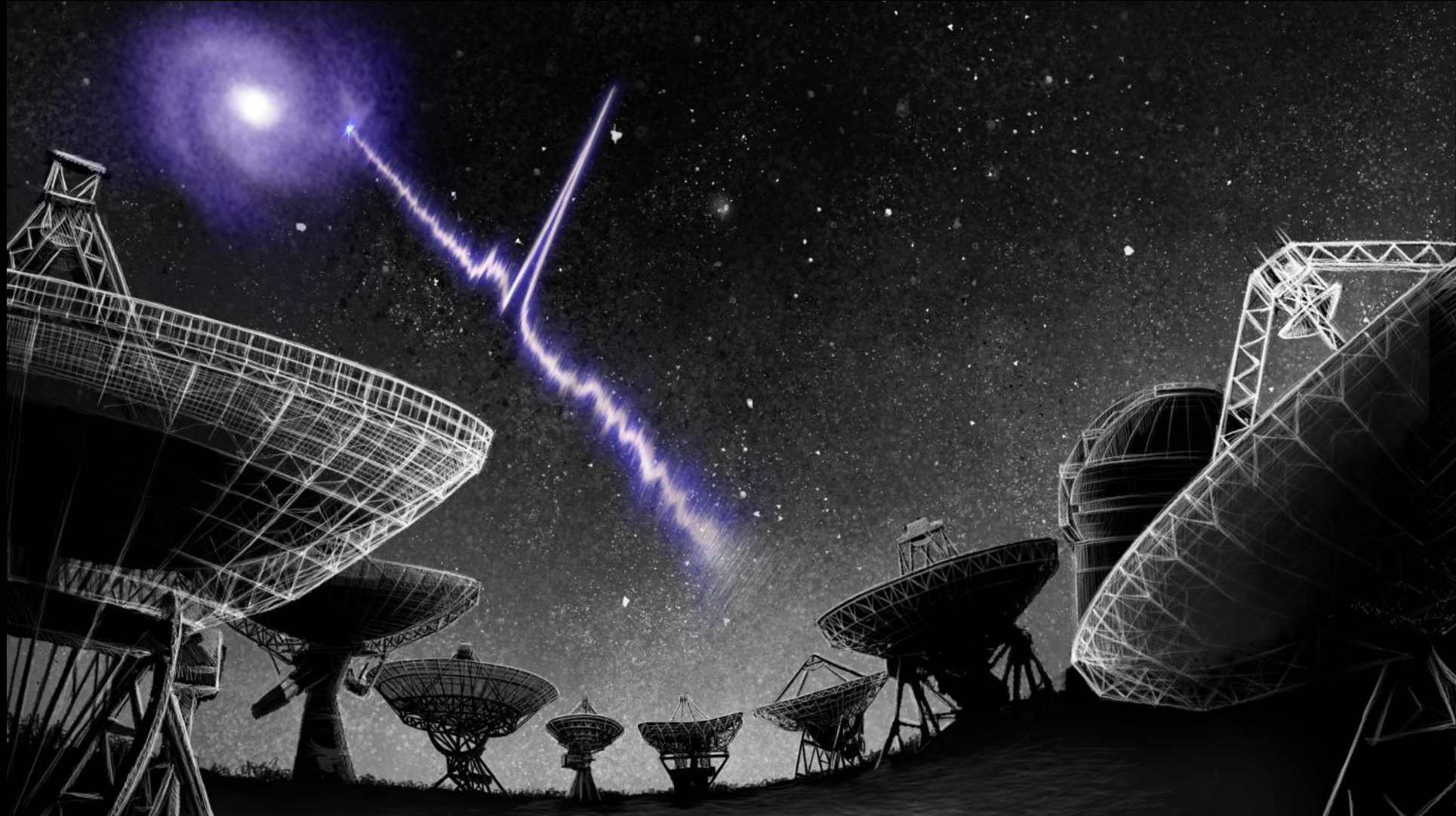
Gogus+2010



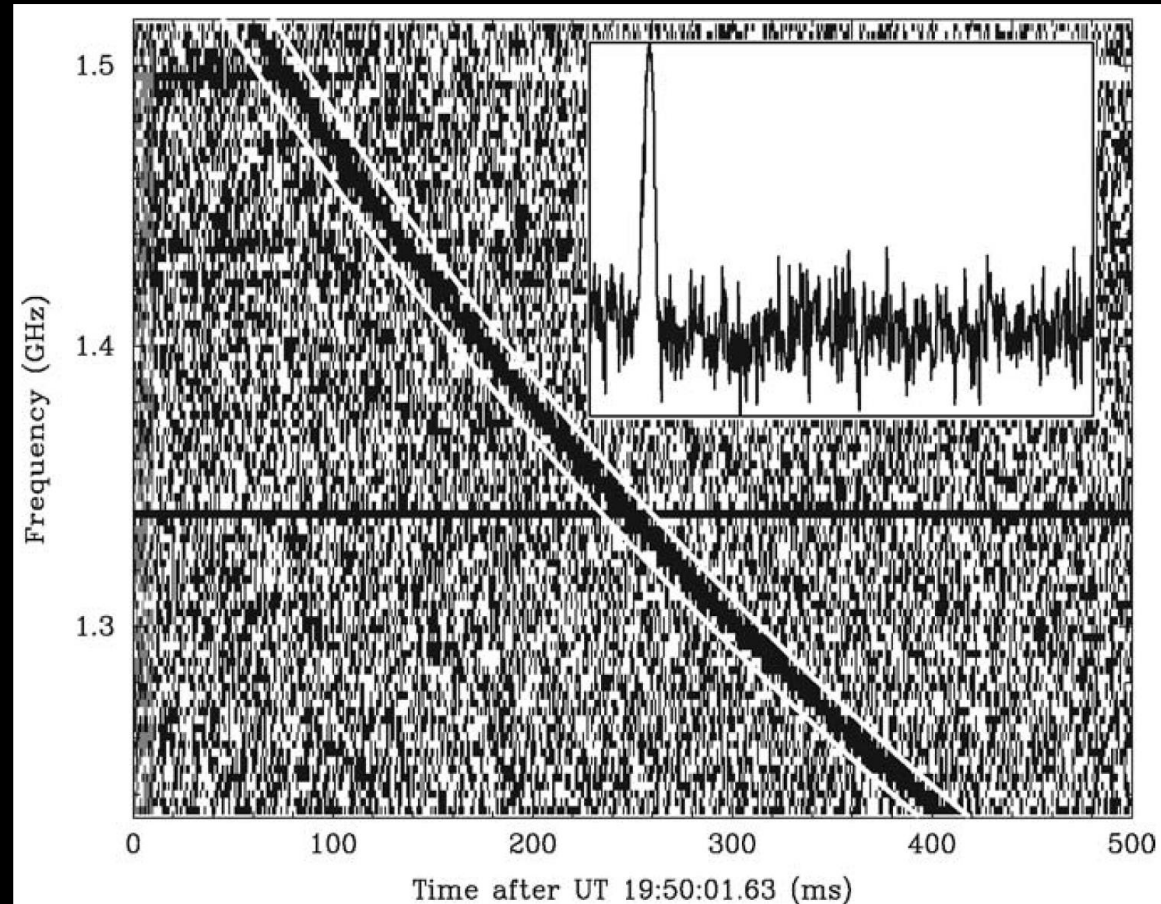
Archibald+2020



Fast Radio Burst, FRB



The Lorimer Burst



Lorimer+2007

- Large dispersion measure – extragalactic, and powerful radio pulses

Dispersion Measure

An EM wave with a frequency of ν emitted from a distance of d . It penetrates a clump of electron plasma with number density of n_e .

The travel time is

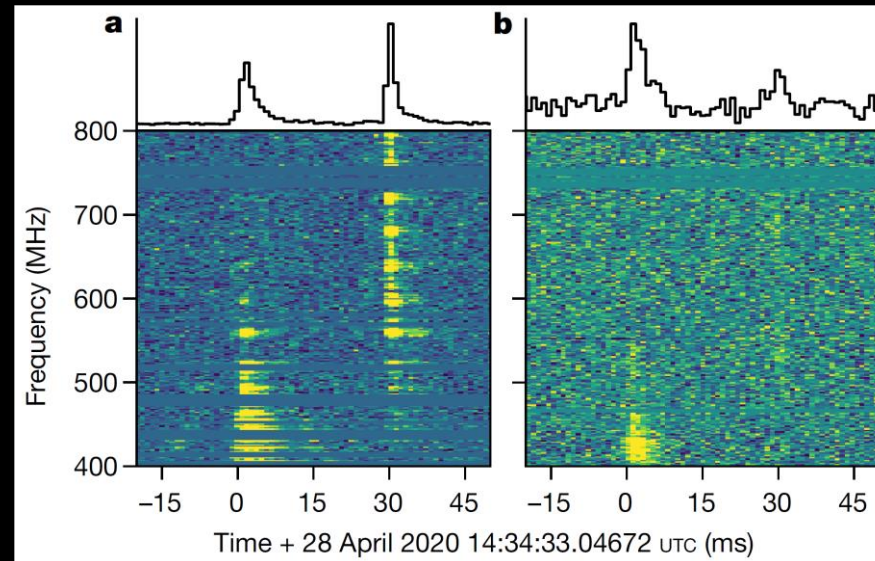
$$t_p = \frac{d}{c} + \frac{e^2}{2\pi m_e c} \frac{\int_0^d n_e dl}{\nu^2}$$

$t_d(\text{delay})$

DM

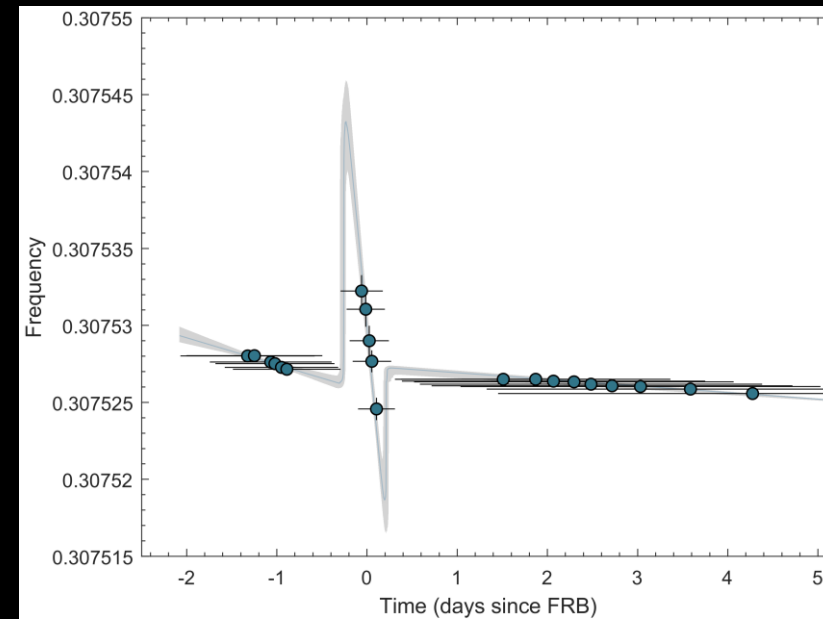
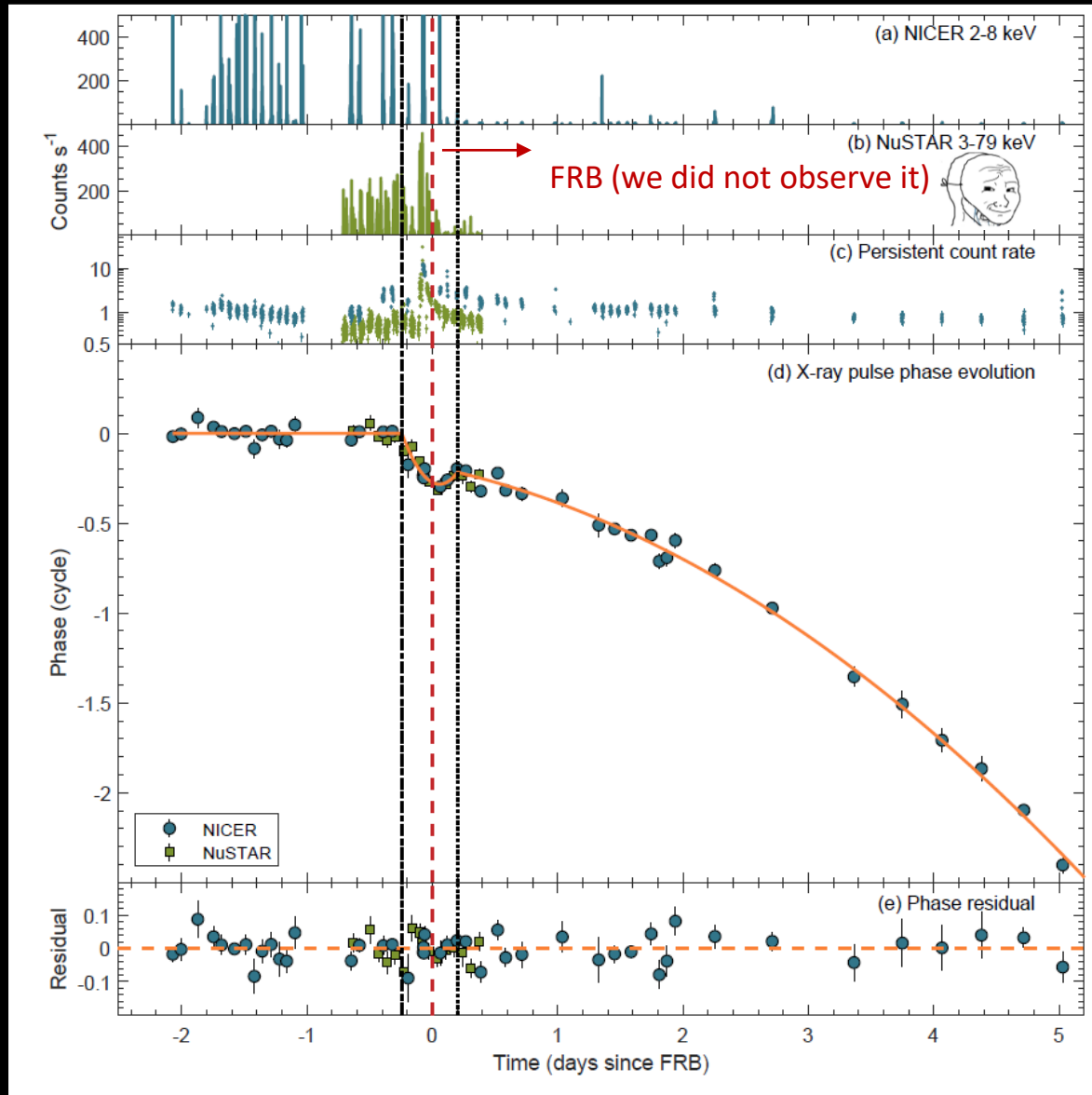
$$t_d = 4140 \left(\frac{DM}{cm^{-3} pc} \right) \left(\frac{\nu}{1 MHz} \right)^{-2} s$$

FRB in the Milky Way



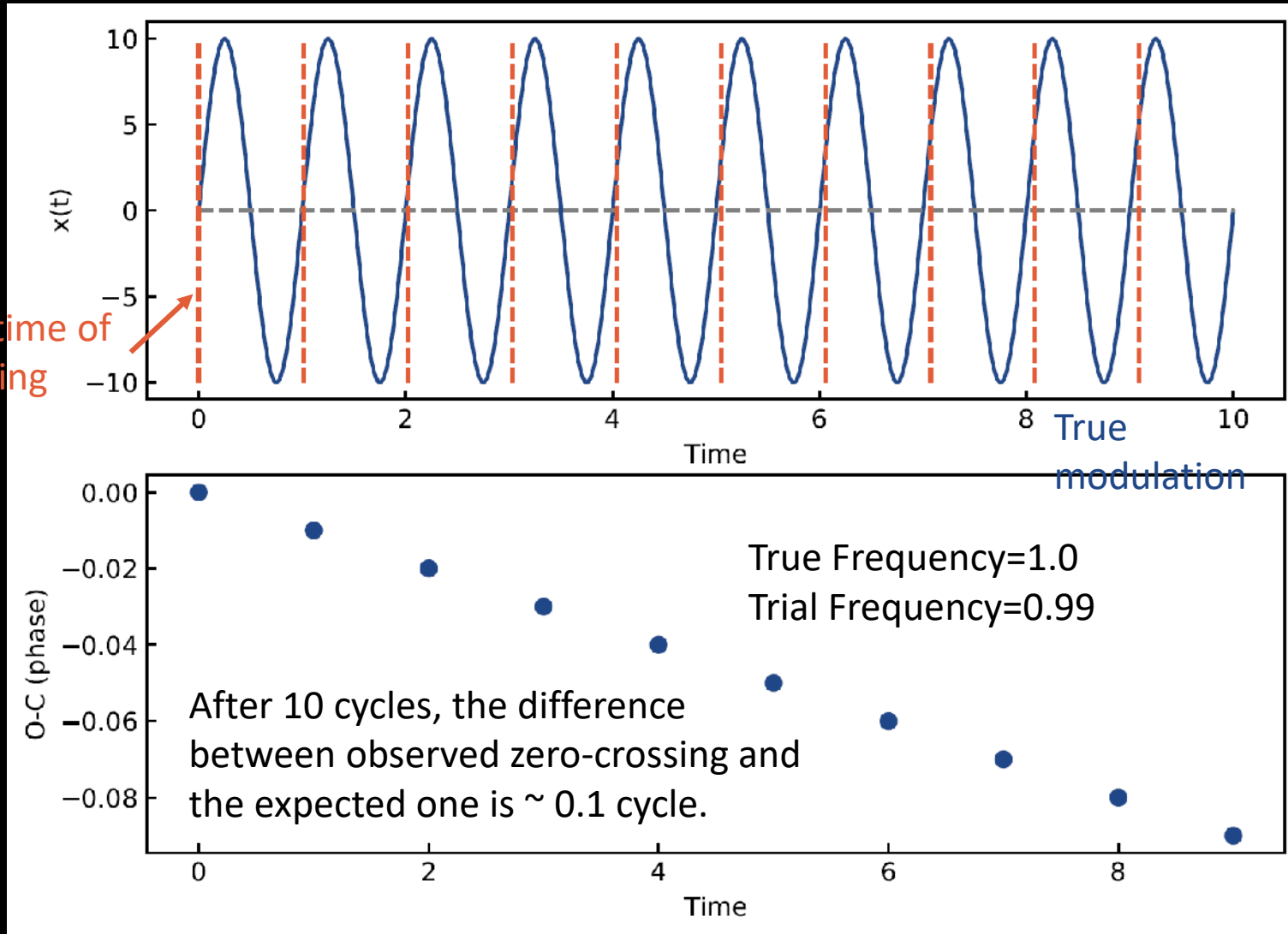
- In 2020, a galactic FRB (200428) was detected in the direction coincide with SGR 1935+2154
 - SGR 1935 is a magnetars and was outbursting
 - Faint end of FRBs
 - Magnetars is one of the origin of FRBs

SGR 1935+2154: 2022 Outburst

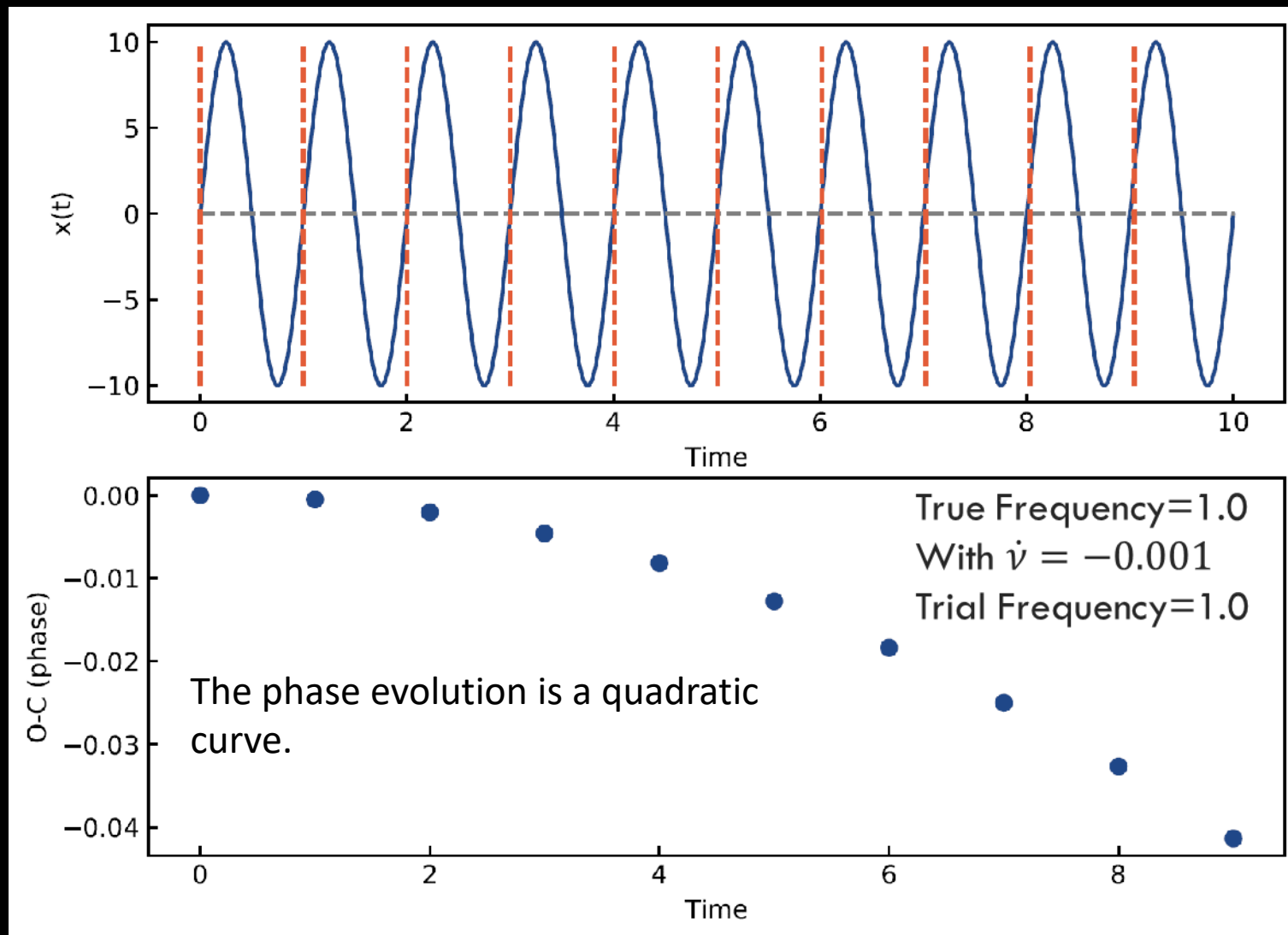


Time-of-Pulse-Arrival Analysis

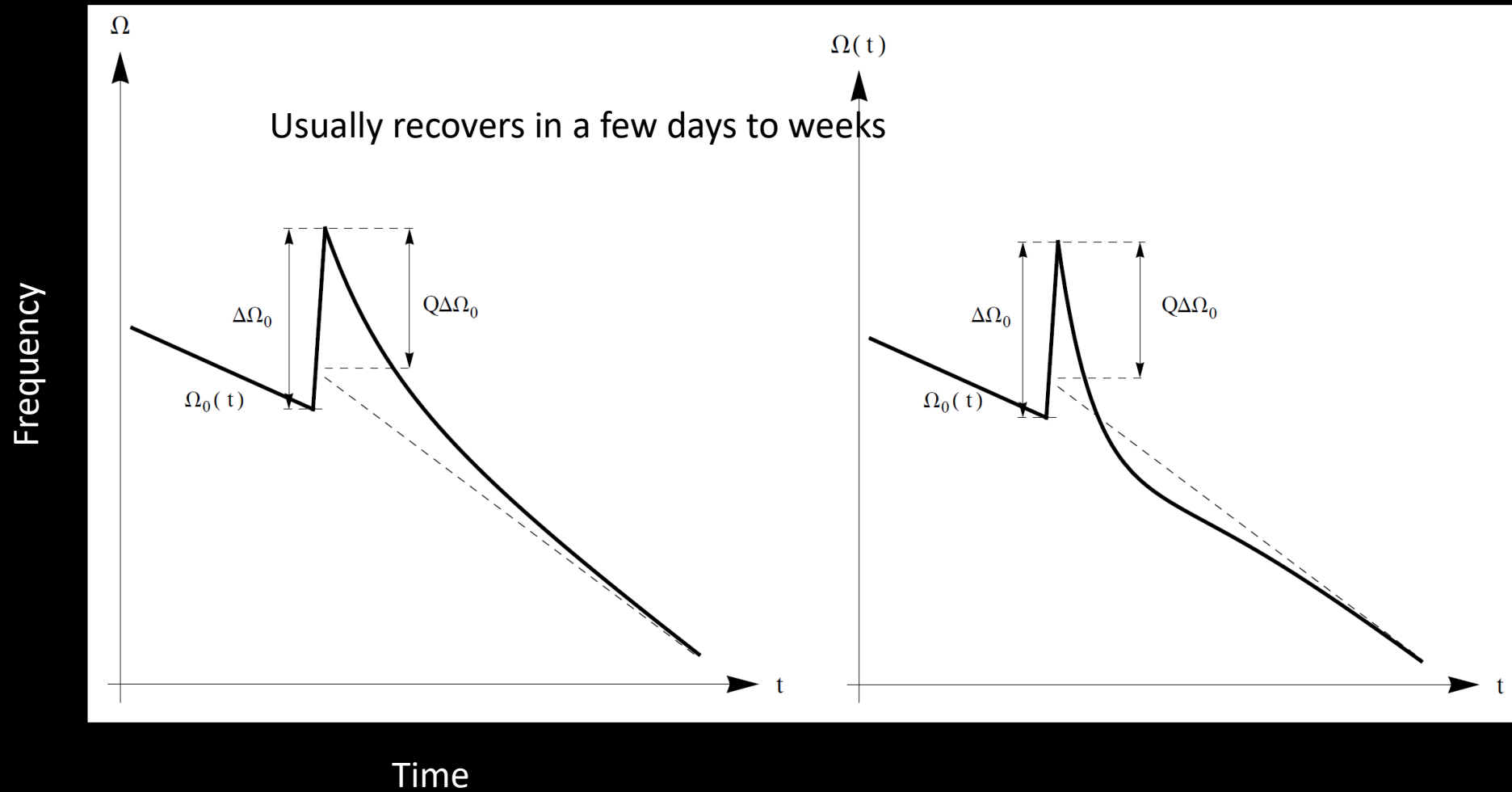
Expected time of zero-crossing



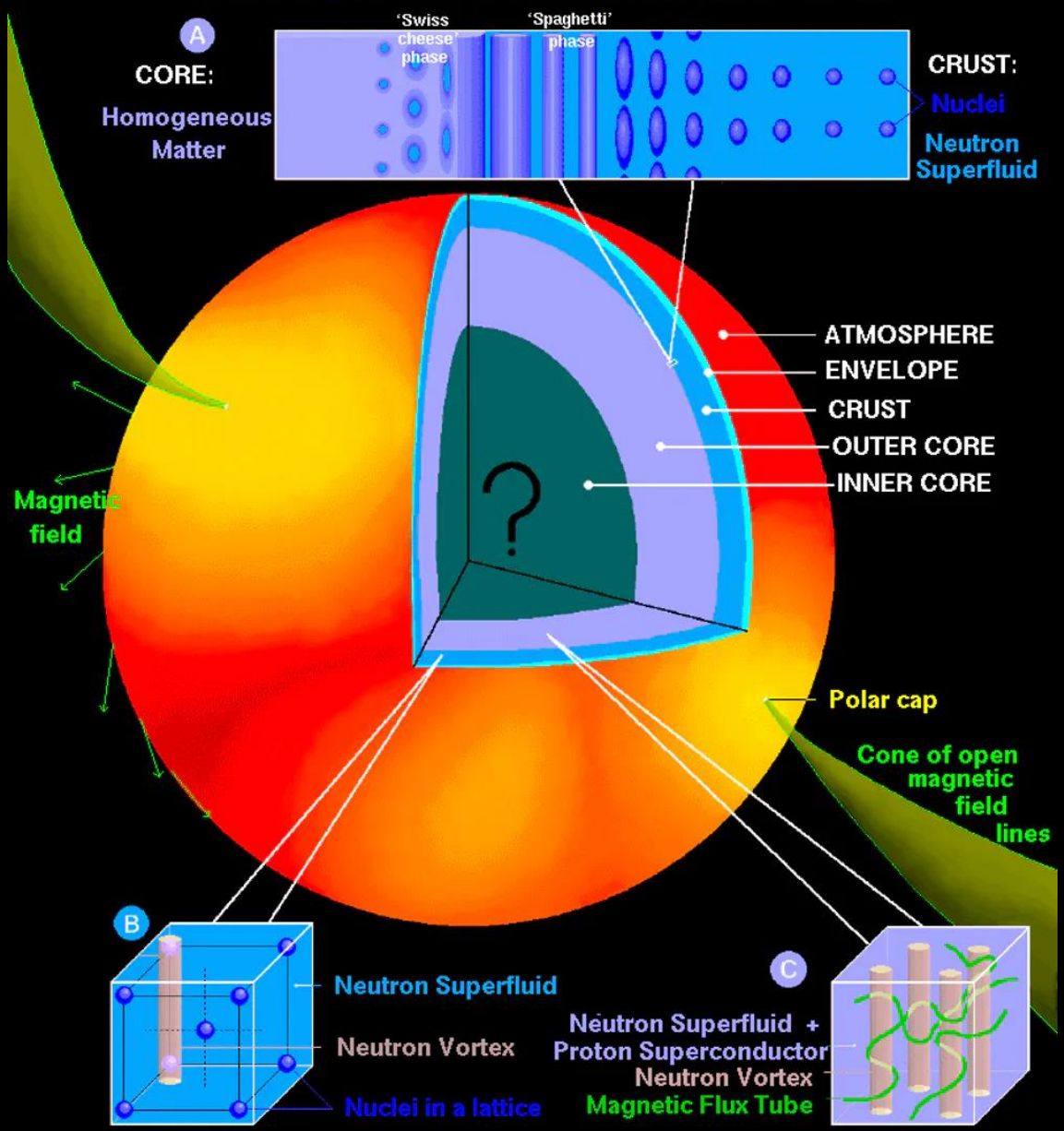
Time-of-Pulse-Arrival Analysis



Glitches: Probe Structures of Neutron Stars

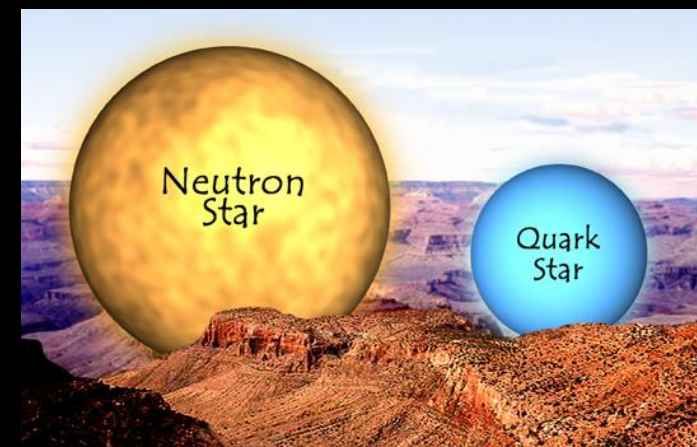
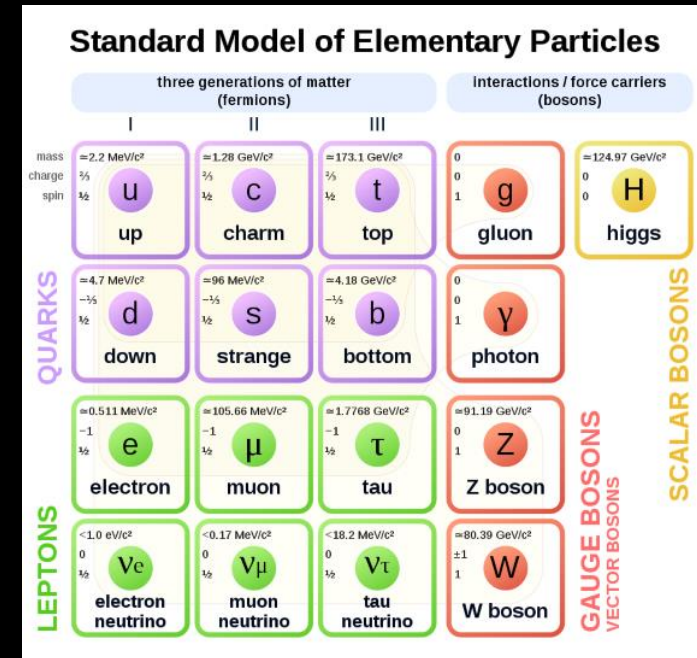


A NEUTRON STAR: SURFACE and INTERIOR



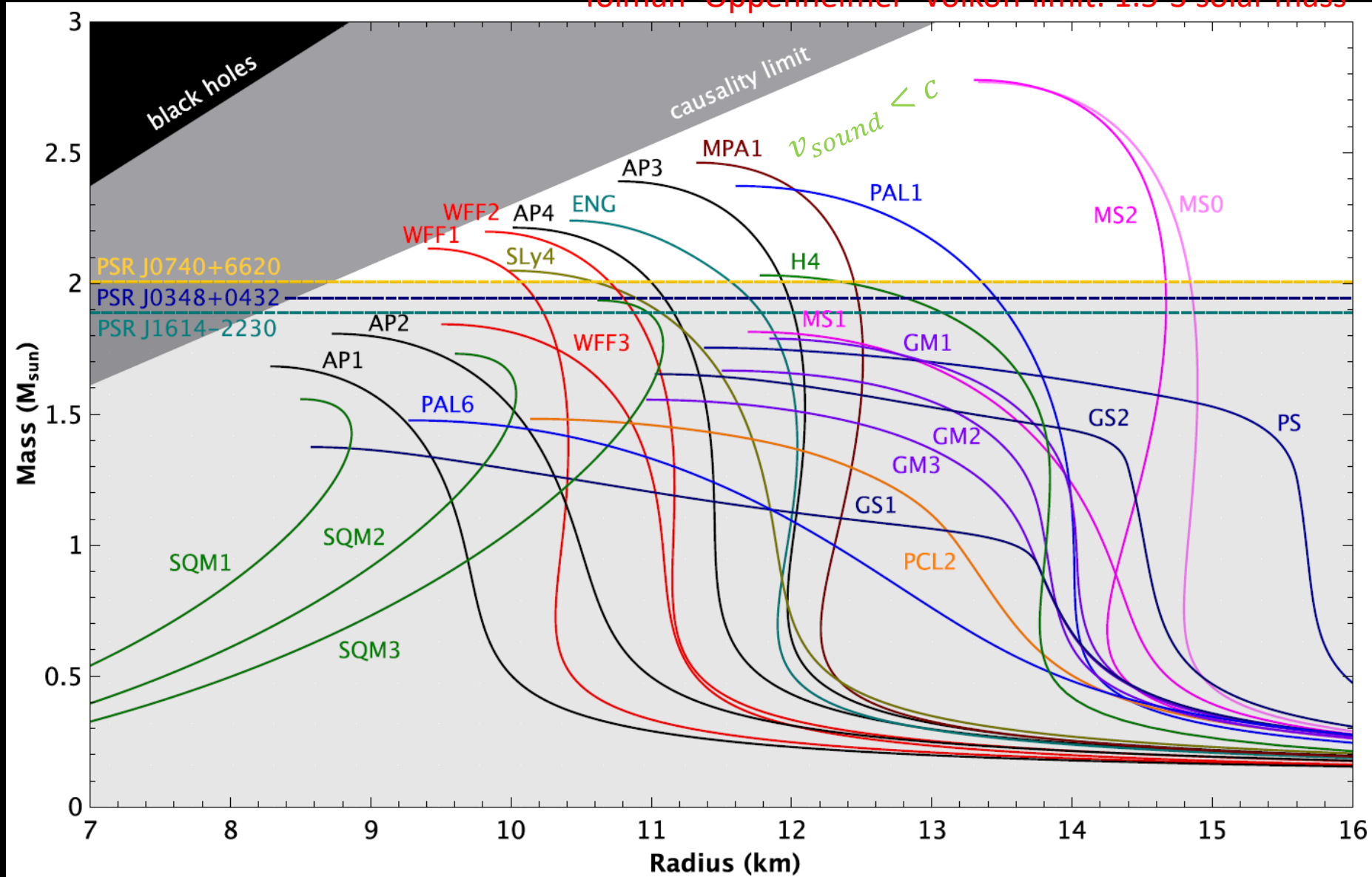
Neutron Stars: Not Only Neutrons

- The radius of a neutron star is roughly 10 km
- The distance between neutrons is about 10^{-15} m
 - Roughly the same as a nucleus
 - Strong force cannot be ignored!
 - We have no idea about the behaviors of these particles under such a high gravity, magnetic field, and temperature.
 - Quark star? Strange star?
 - Neutron stars are natural laboratories for particle physics.



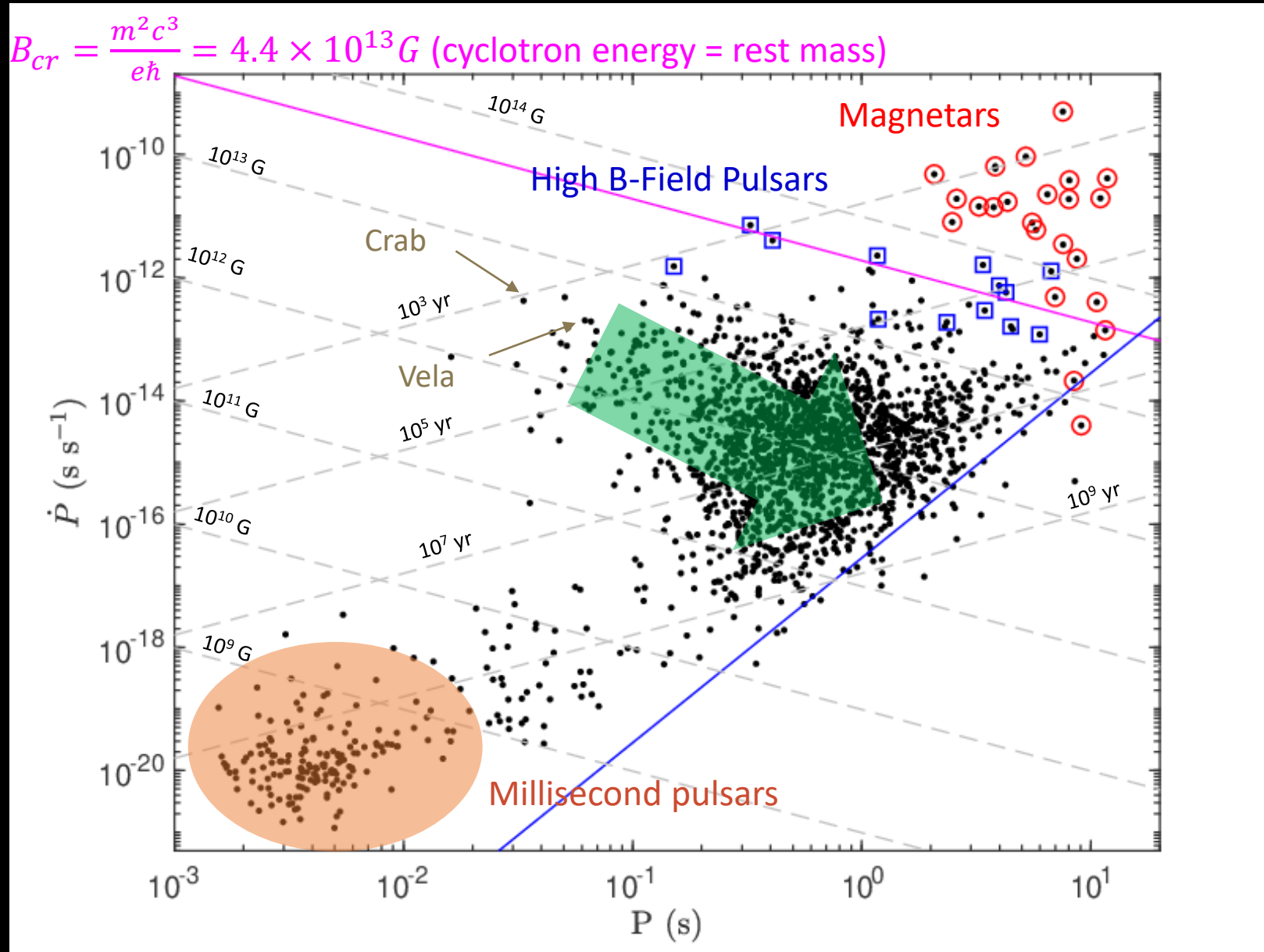
Mass-Radius Relation

Tolman–Oppenheimer–Volkoff limit: 1.5-3 solar mass

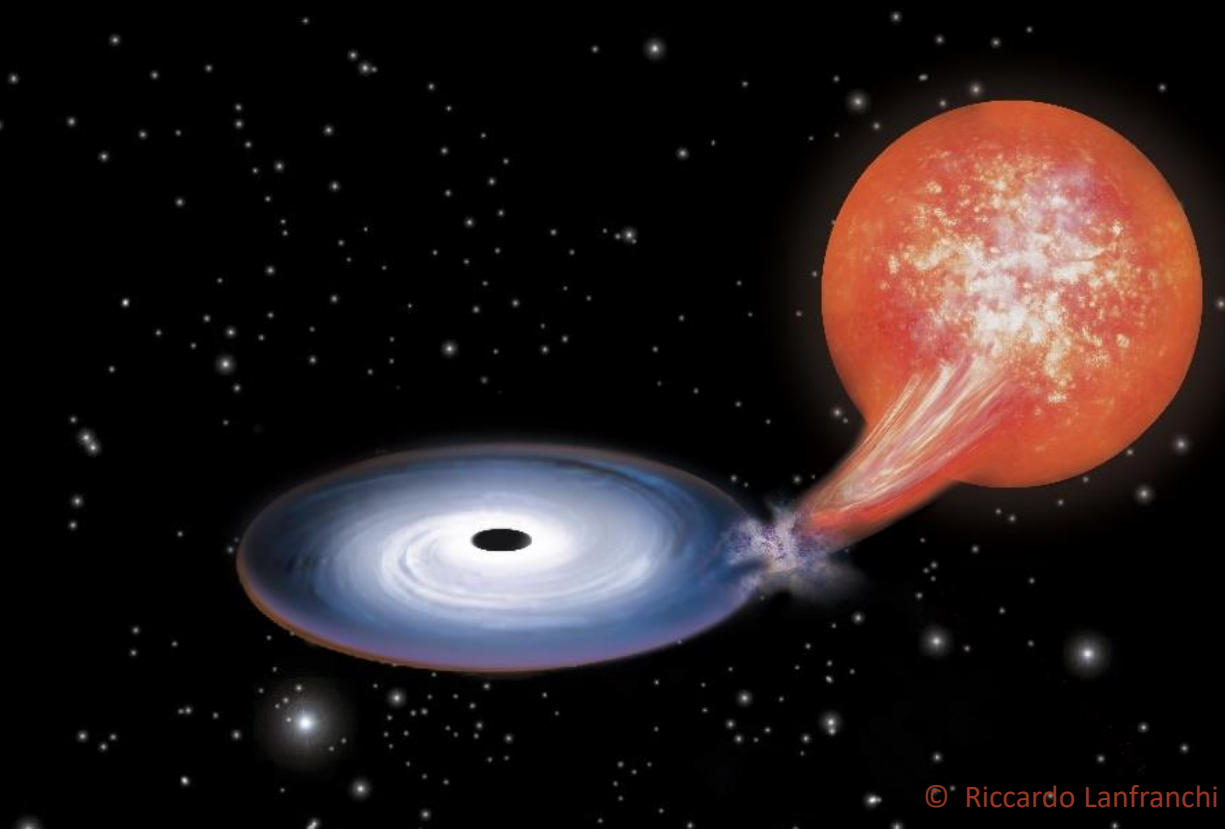


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P-Pdot Diagram



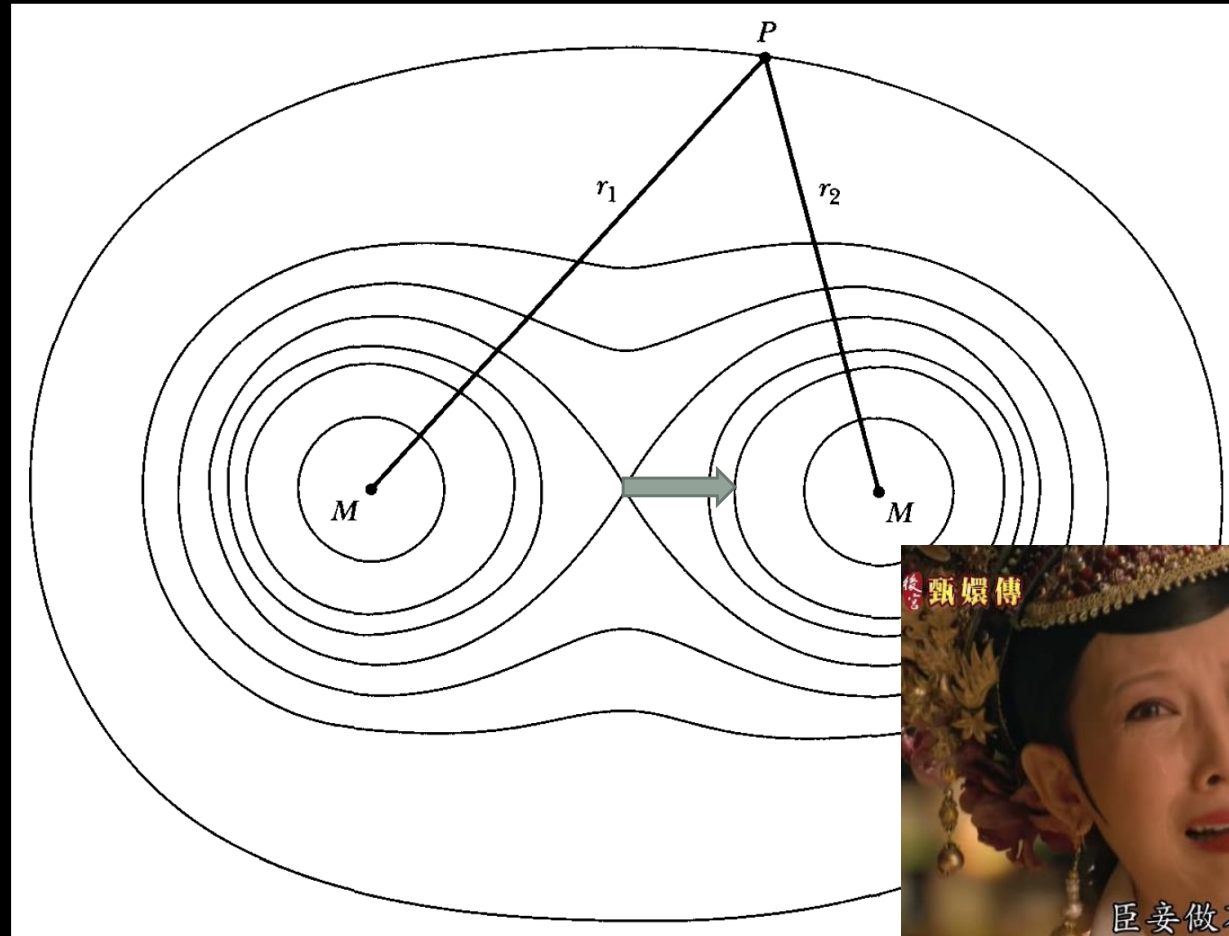
Compact Objects in Binary Systems



© Riccardo Lanfranchi

- Energy source: gravitational potential energy
 - $\dot{E} = \eta \dot{m} c^2$ ($\eta \gtrsim 0.1$)
 - X-ray binary systems (because they emitted strong X-rays)

Equipotential Surface

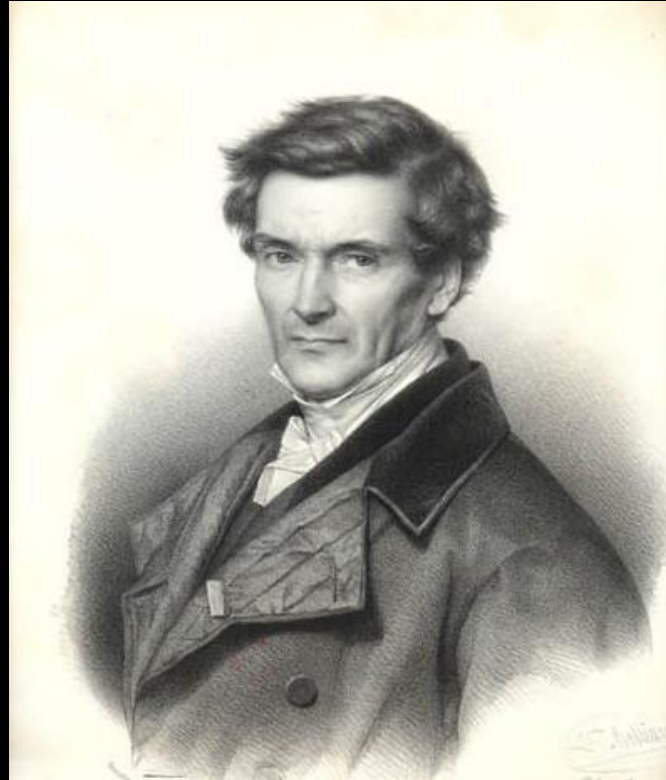


Because two stars orbiting with each other, we have to consider...

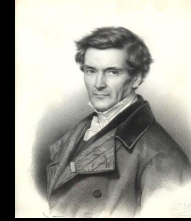
柯氏力



Coriolis Force



Coriolis Force



Noninertial Reference Frame

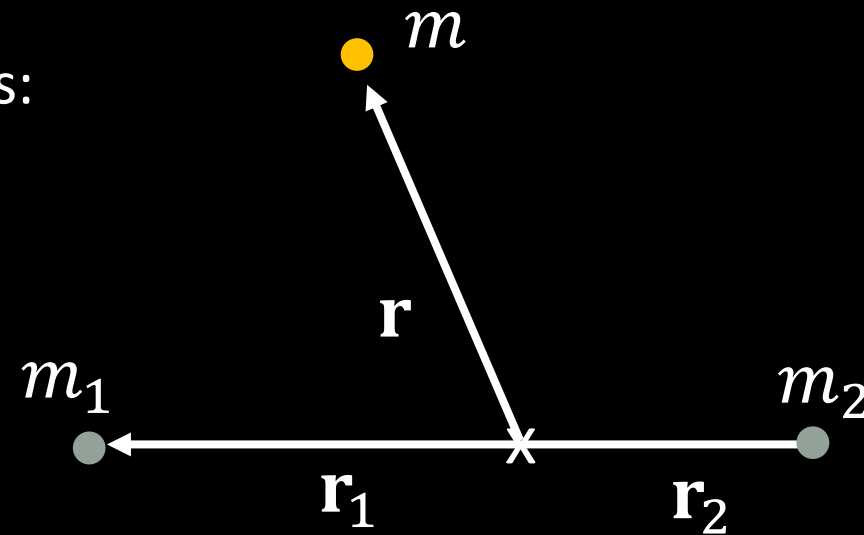
The force \mathbf{F}_m on a mass m at position \mathbf{r} in the rotating reference frame contains several terms:

$$\mathbf{F}_m = -\frac{Gm_1m}{|\mathbf{r} - \mathbf{r}_1|^3} (\mathbf{r} - \mathbf{r}_1)$$

$$-\frac{Gm_1m}{|\mathbf{r} - \mathbf{r}_2|^3} (\mathbf{r} - \mathbf{r}_2)$$

$$-m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \quad \dots \text{Centrifugal force} = +m\omega^2 \mathbf{r}$$

$$-2m\boldsymbol{\omega} \times \dot{\mathbf{r}} \quad \dots \text{Coriolis force}$$



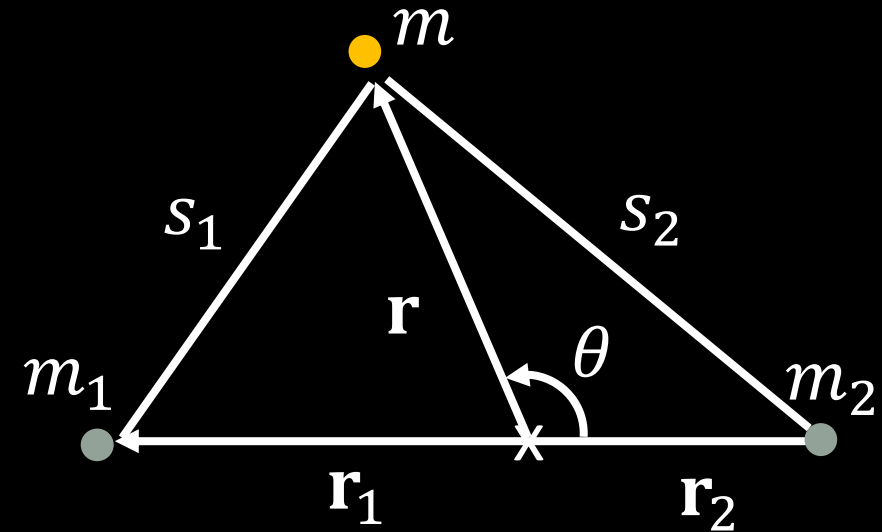
Noninertial Reference Frame

The effective potential can then be derived as

$$\Phi = -G \left(\frac{m_1}{|\mathbf{r} - \mathbf{r}_1|} + \frac{m_2}{|\mathbf{r} - \mathbf{r}_2|} \right) - \frac{1}{2} \omega^2 |\mathbf{r}|^2$$

From Newtonian dynamics, we know

$$\omega^2 = \frac{G(m_1 + m_2)}{(r_1 + r_2)^3} = \frac{G(m_1 + m_2)}{a^3}$$



If we choose CM as the origin

$$\mathbf{r}_1 = -\frac{m_2}{m_1 + m_2} a \hat{x}$$

$$\mathbf{r}_2 = \frac{m_1}{m_1 + m_2} a \hat{x}$$

Noninertial Reference Frame

We can define a mass ratio $q \equiv \frac{m_2}{m_1}$

Then

$$\omega^2 = \frac{G(m_1 + m_2)}{a^3} = \frac{Gm_1(1 + q)}{a^3}$$

$$\mathbf{r}_1 = -\frac{m_2}{m_1 + m_2} a \hat{x} = -\frac{q}{1 + q} a \hat{x}$$

$$\mathbf{r}_2 = \frac{m_1}{m_1 + m_2} a \hat{x} = \frac{1}{1 + q} a \hat{x}$$

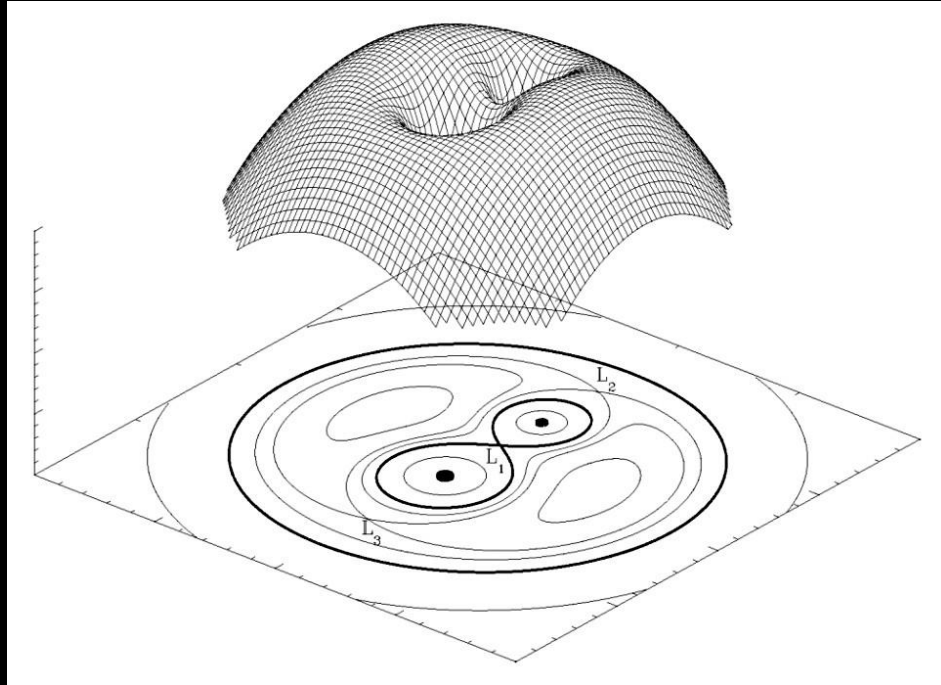
Noninertial Reference Frame

The effective potential can then be written as

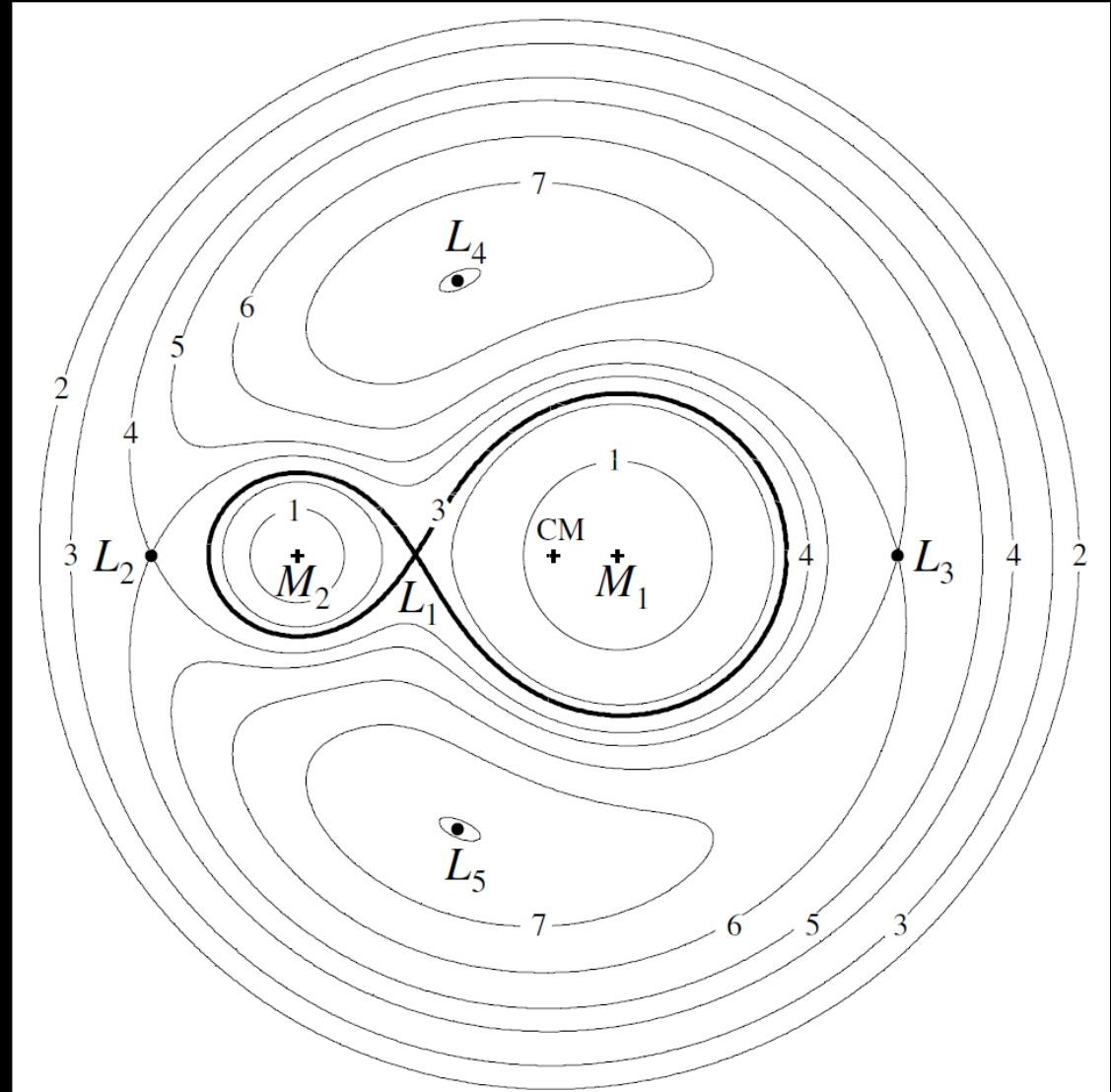
$$\begin{aligned}\Phi &= -G \left(\frac{m_1}{|\mathbf{r} - \mathbf{r}_1|} + \frac{m_2}{|\mathbf{r} - \mathbf{r}_2|} \right) - \frac{1}{2} \omega^2 |\mathbf{r}|^2 \\ &= -\frac{Gm_1}{a} \left(\frac{1}{\left| \frac{\mathbf{r}}{a} + \frac{q}{1+q} \hat{\mathbf{x}} \right|} + \frac{q}{\left| \frac{\mathbf{r}}{a} - \frac{1}{1+q} \hat{\mathbf{x}} \right|} \right) \\ &= -\frac{Gm_1(1+q)}{2a} \left[\left| \frac{\mathbf{r}}{a} \right|^2 - \left(\frac{\mathbf{r}}{a} \cdot \hat{\mathbf{z}} \right)^2 \right] = -\frac{Gm_1}{a} f\left(\frac{\mathbf{r}}{a}, q\right)\end{aligned}$$

The “shape” of the equipotential surface only depends on the mass ratio q

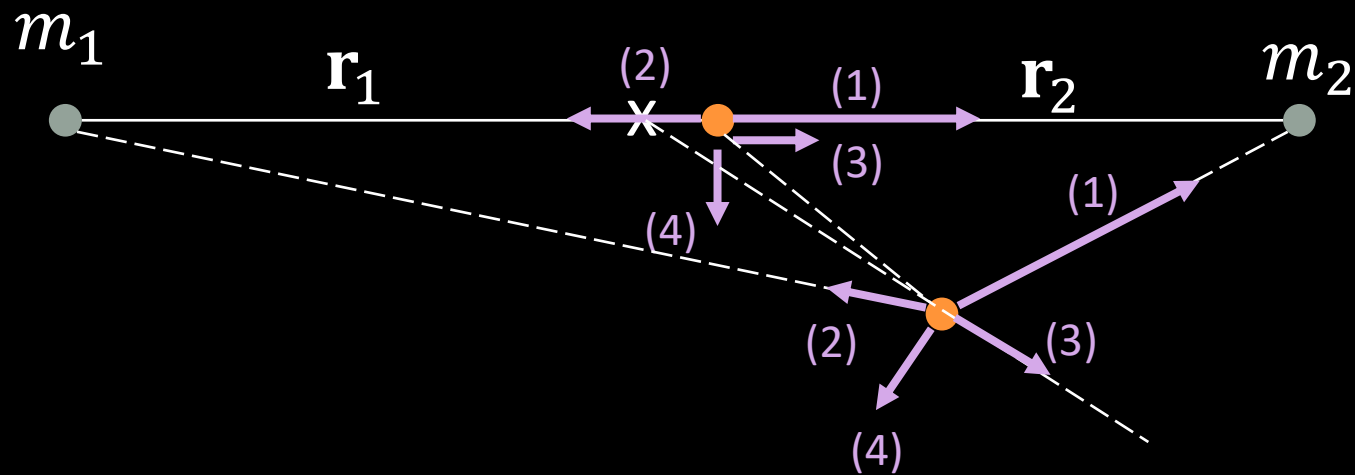
Roche Lobe and Lagrange Points



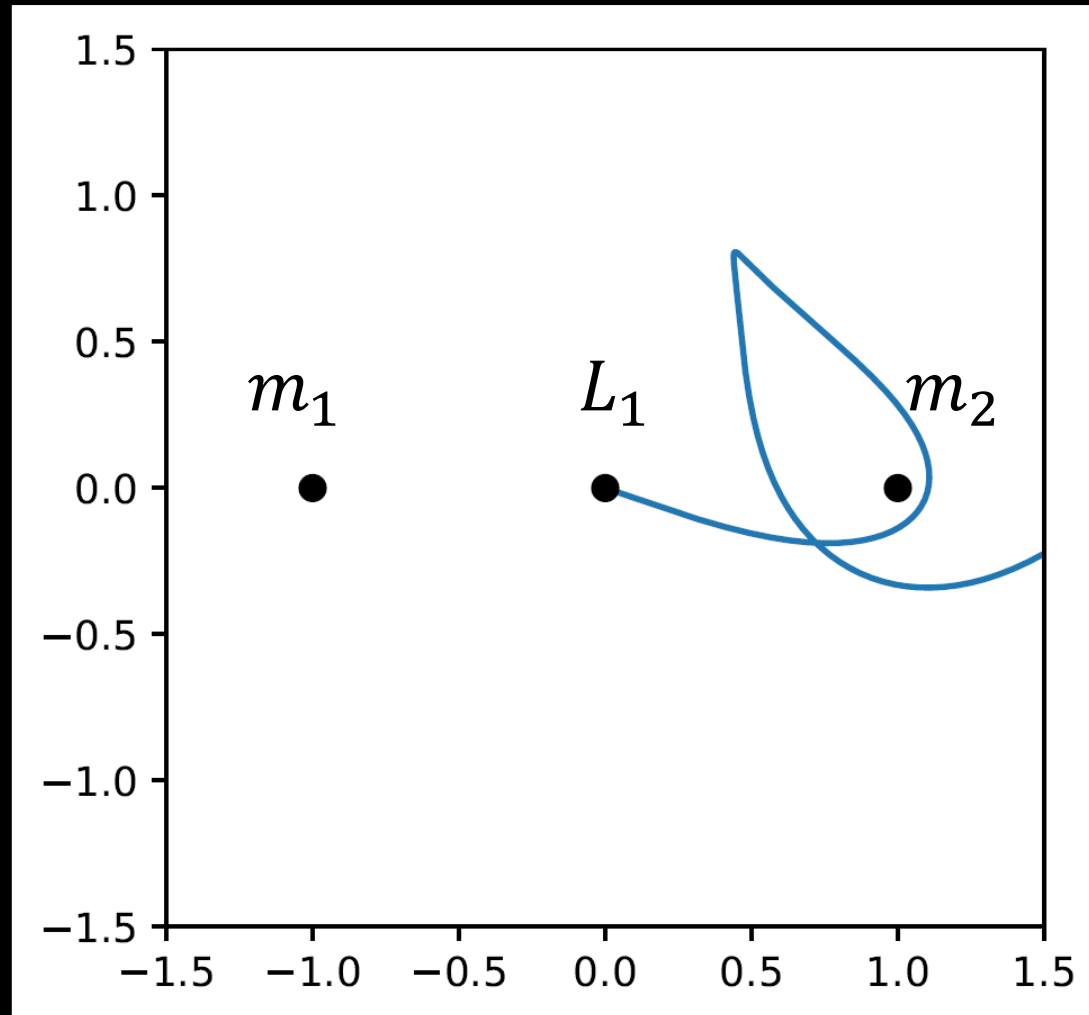
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$$\mathbf{F}_m = -\frac{Gm_1m}{|\mathbf{r} - \mathbf{r}_1|^3}(\mathbf{r} - \mathbf{r}_1) - \frac{Gm_1m}{|\mathbf{r} - \mathbf{r}_2|^3}(\mathbf{r} - \mathbf{r}_2) \\ -m\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2m\boldsymbol{\omega} \times \dot{\mathbf{r}}$$



Accretion Trajectory



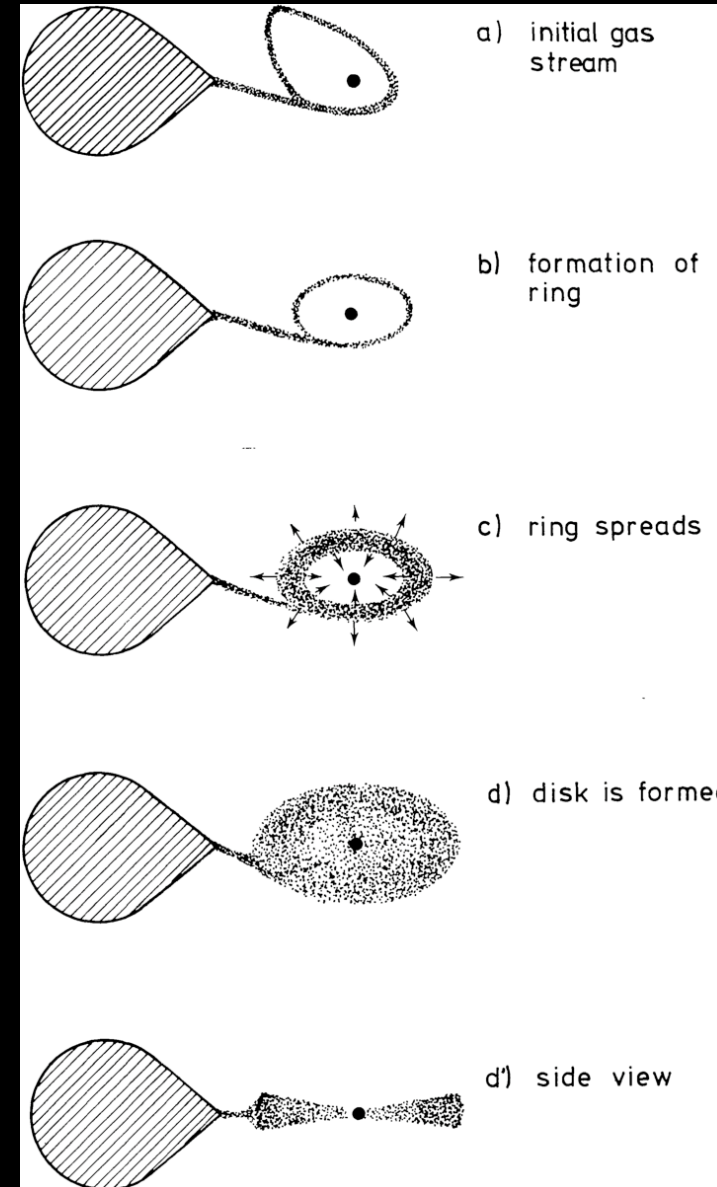
Accretion Disk

The trajectory will gradually become circular and form a ring due to viscosity, which dissipates the dynamic energy into heat.

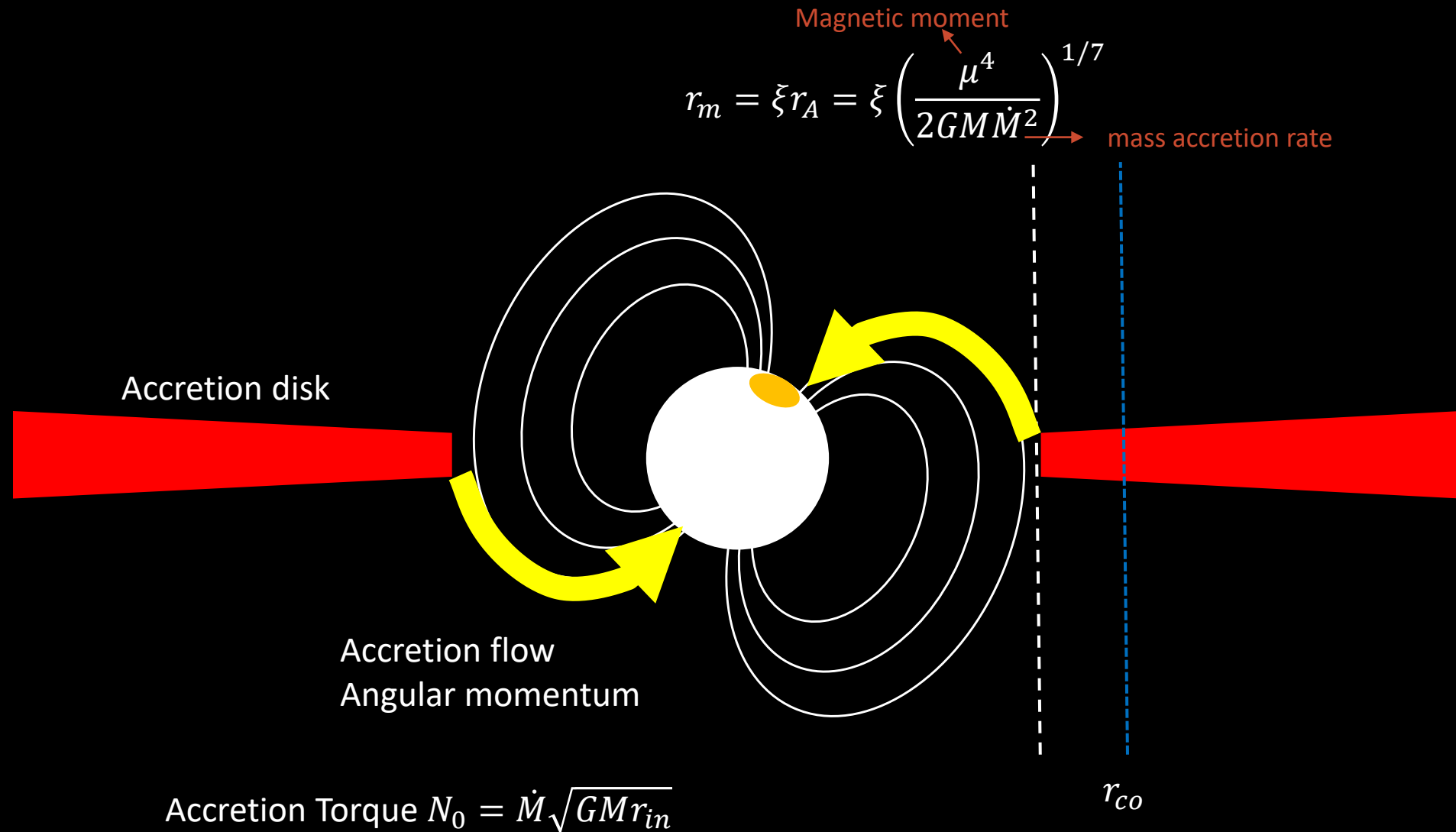
Viscosity: friction force between adjunct layers with different flow velocities

The different Keplerian velocity induces the viscosity force in the ring, and diffuse the ring into a disk.

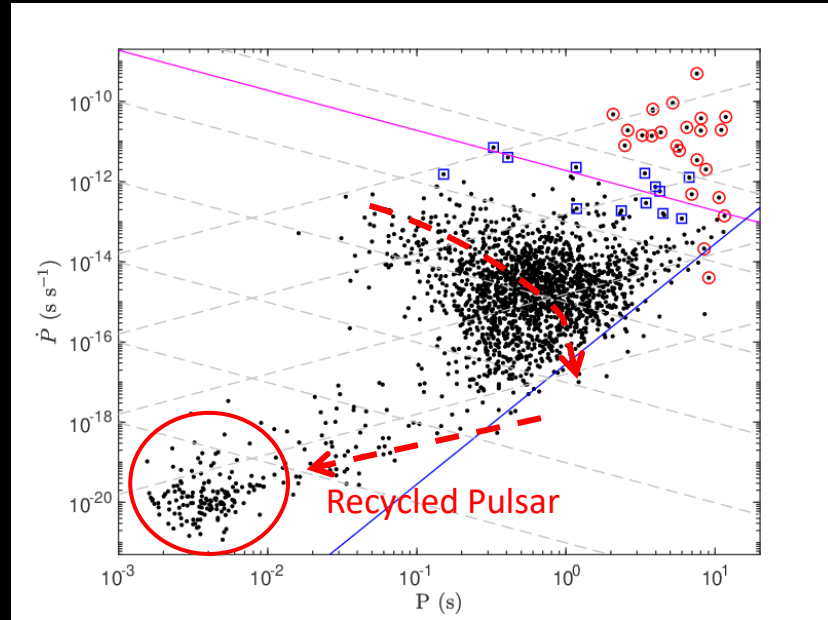
Lose energy \rightarrow emission



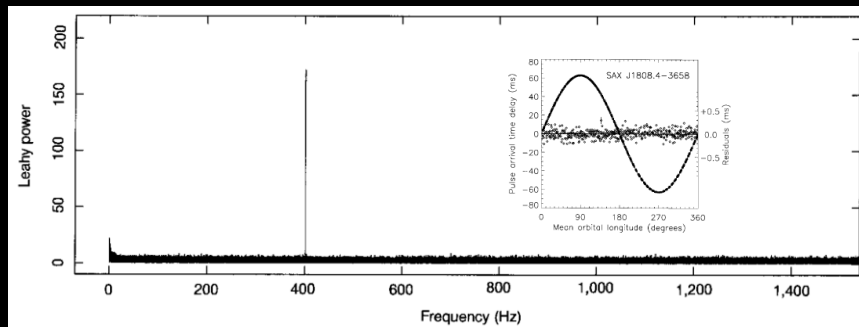
Neutron star: spin-up!



Millisecond Pulsars



© NASA



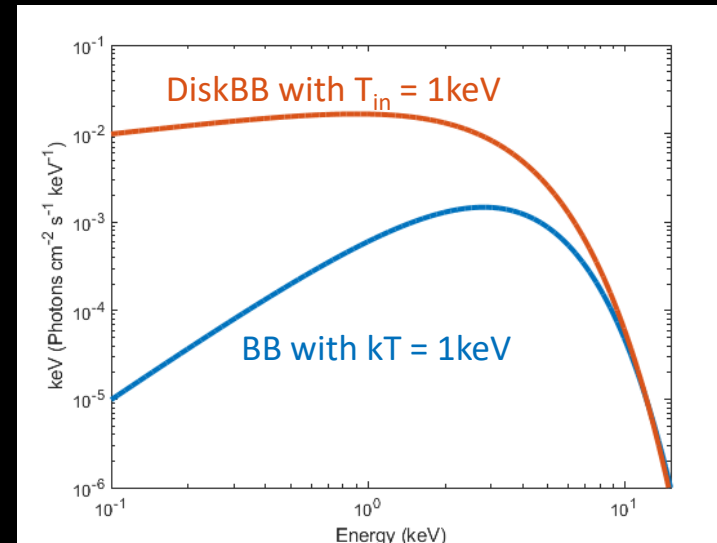
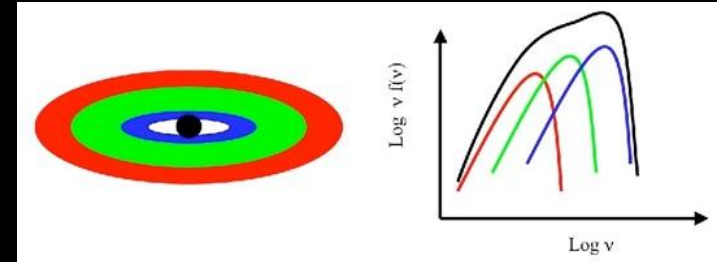
First evidence: MSP in LMXBs (Wijnands+1998)

How about black holes?

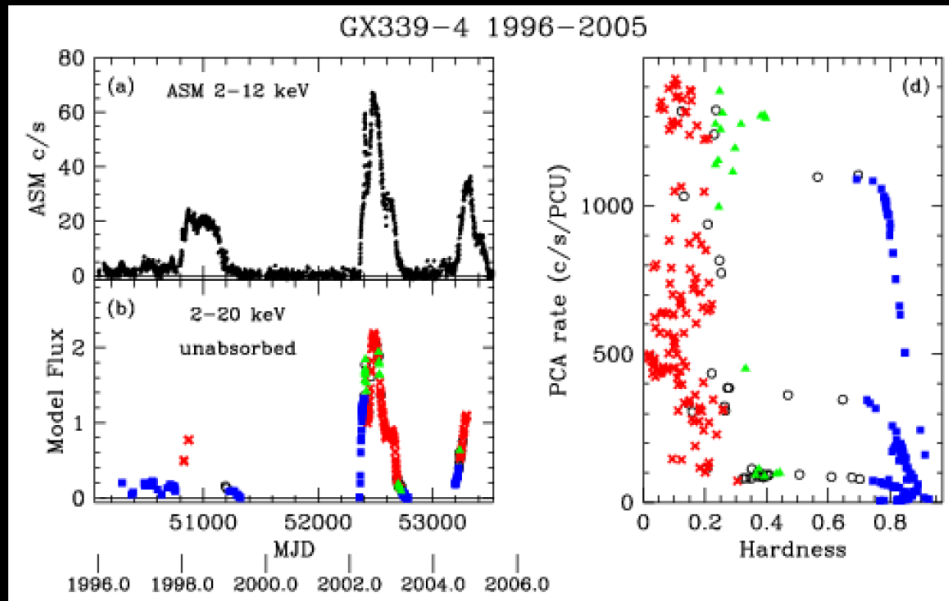
- Unlike neutron stars, black holes have no pulsed radio/X-ray emission
 - No type-I X-ray burst as well.
- Black holes can drain materials from companion stars or nearby interstellar mediums, and emit radiations through accretion process.
- They are ideal targets to study physics of accretion disks and to test general relativity.

Disk Black Body

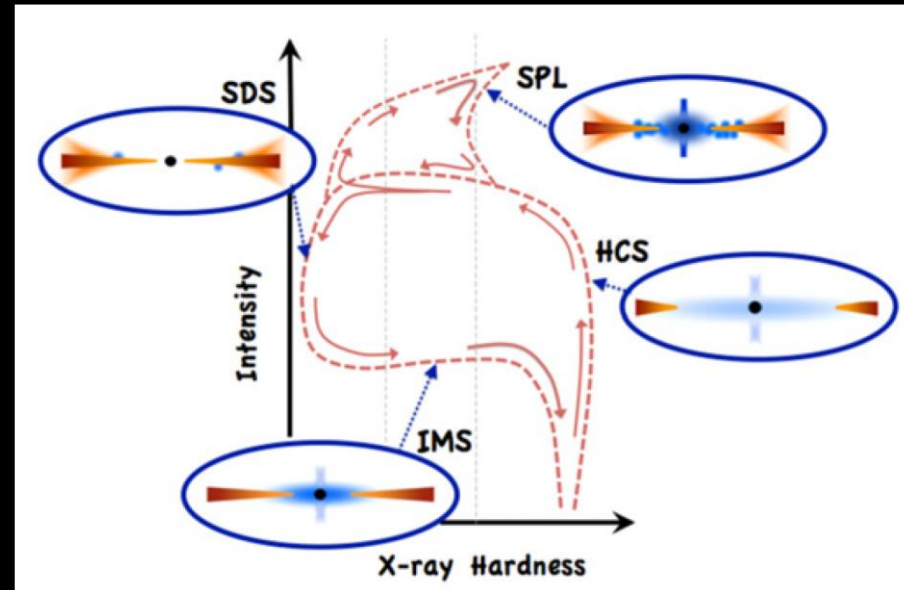
- The accretion disk has a blackbody-like spectrum in X-rays
 - Usually seen in black hole X-ray binaries and active galactic nuclei
- The temperature increases toward the central objects
 - $T_R \propto R^{-0.75}$
- The spectrum can be modeled as superposition of multi-temperature blackbodies



Soft X-ray Transients



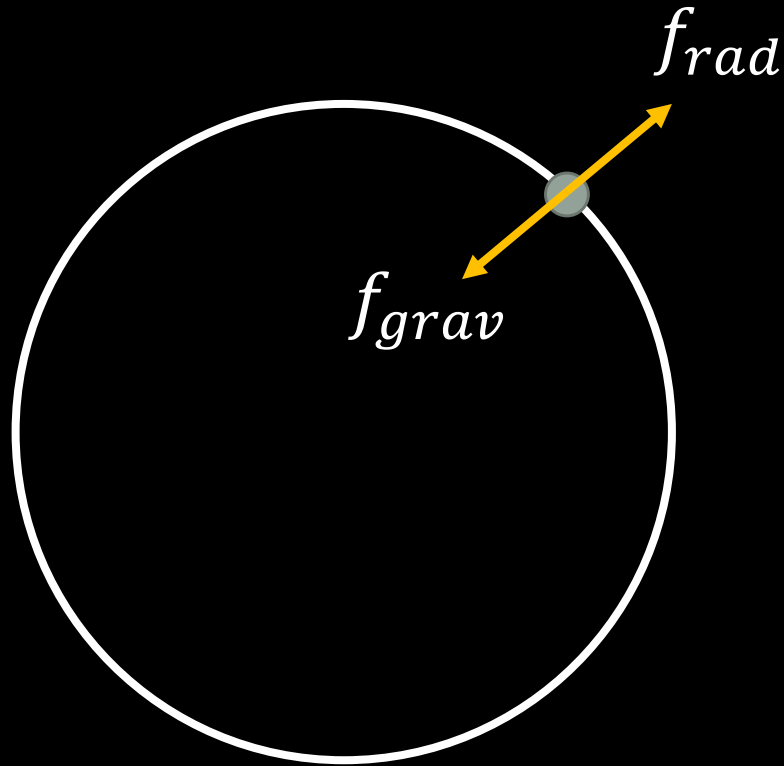
Remillard (2004)



Tetarenko et al. (2016)

- Hardness: count rate ratio between hard and soft X-rays
- In general, the peak luminosity of an outburst cannot exceed the Eddington luminosity

Eddington Limit



(Assumption: Thomson scattering)

$$f_{rad} = \frac{\kappa L}{4\pi cr^2}$$

κ : opacity

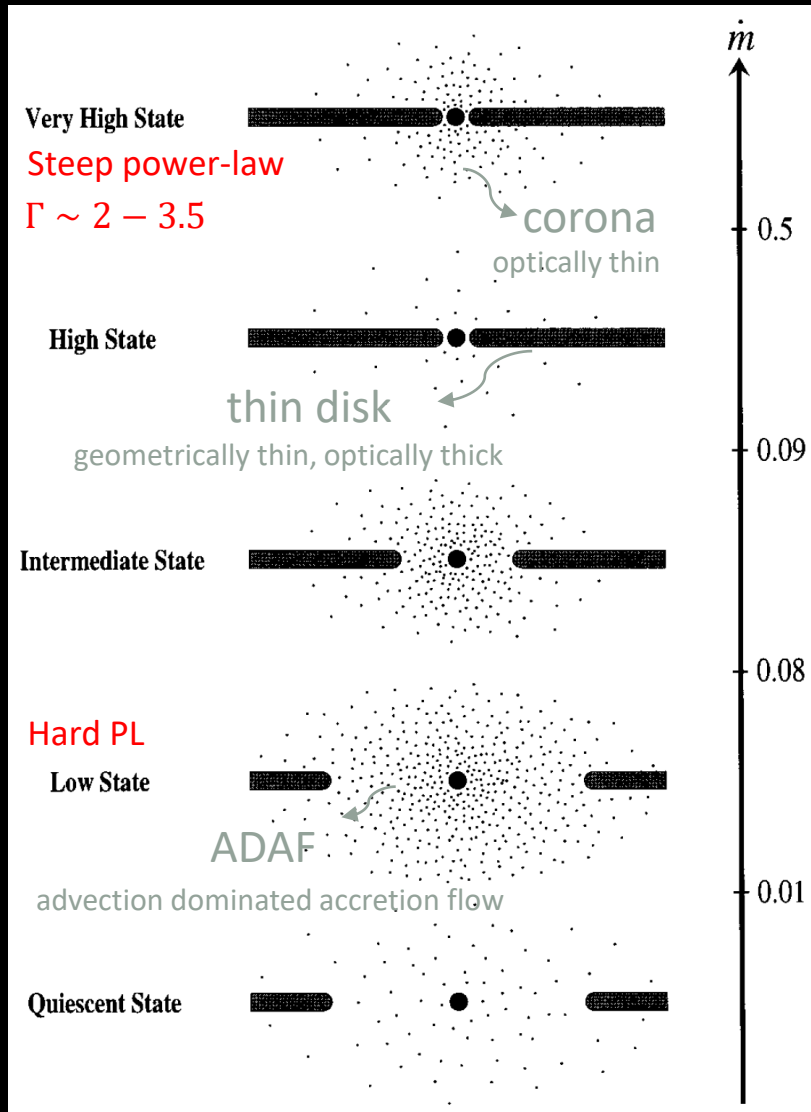
$$f_{grav} = \frac{GM}{r^2}$$

$$\Rightarrow L_{Edd} = \frac{4\pi cGM}{\kappa}$$

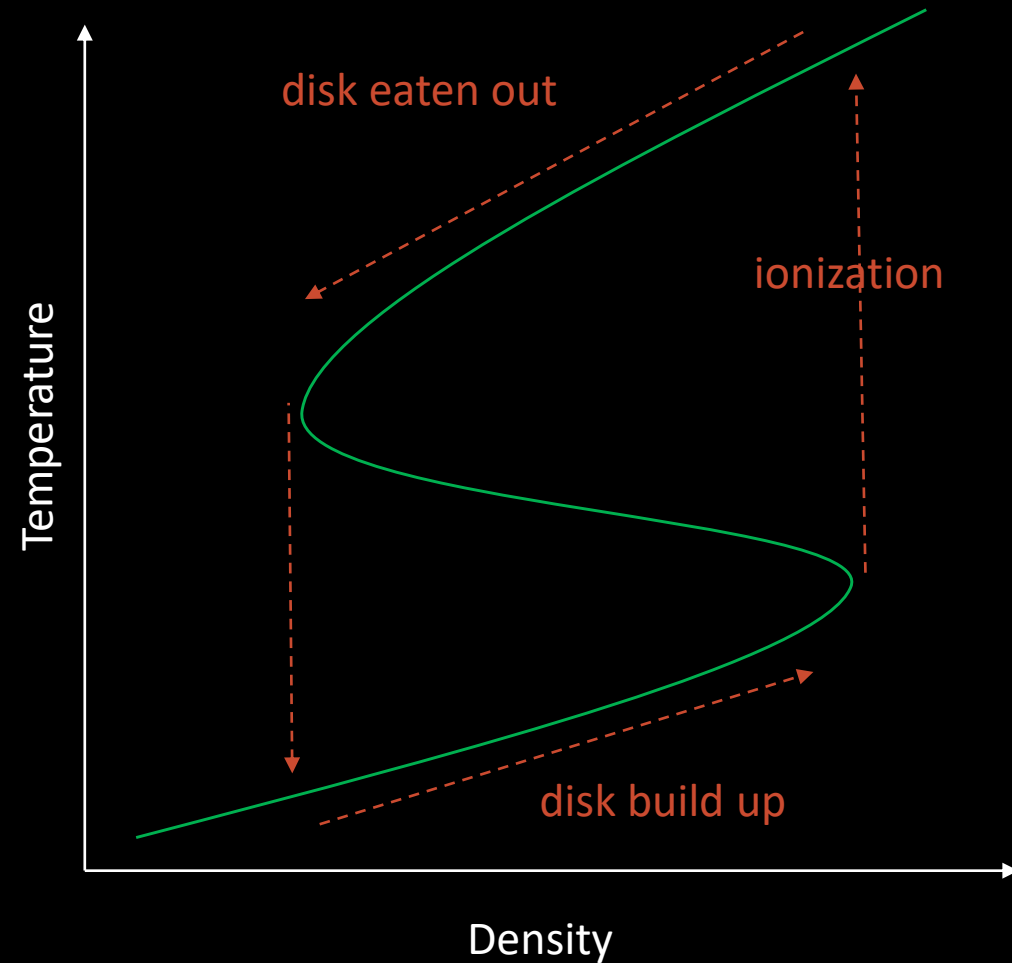
$$= 1.3 \times 10^{38} \left(\frac{M}{M_{\odot}} \right) \text{ erg s}^{-1}$$

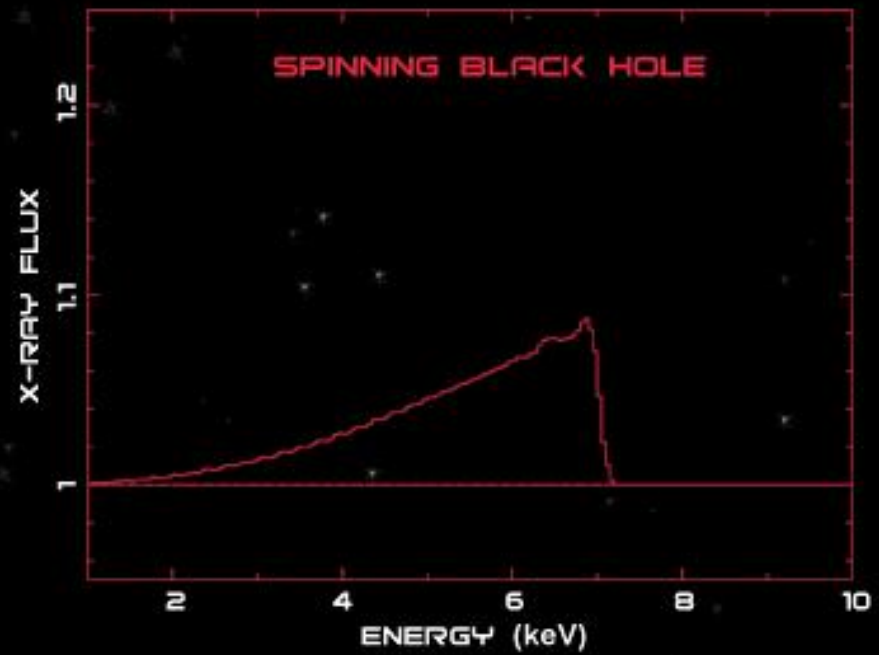
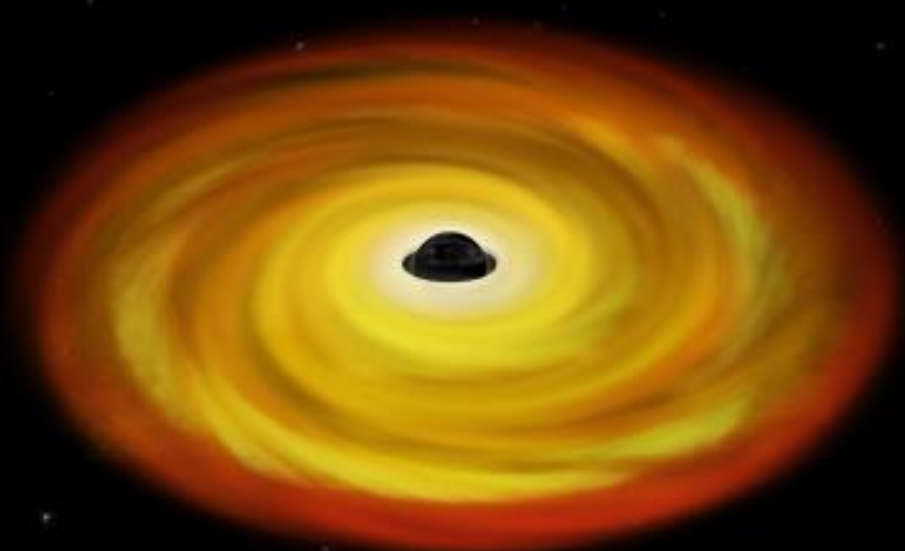
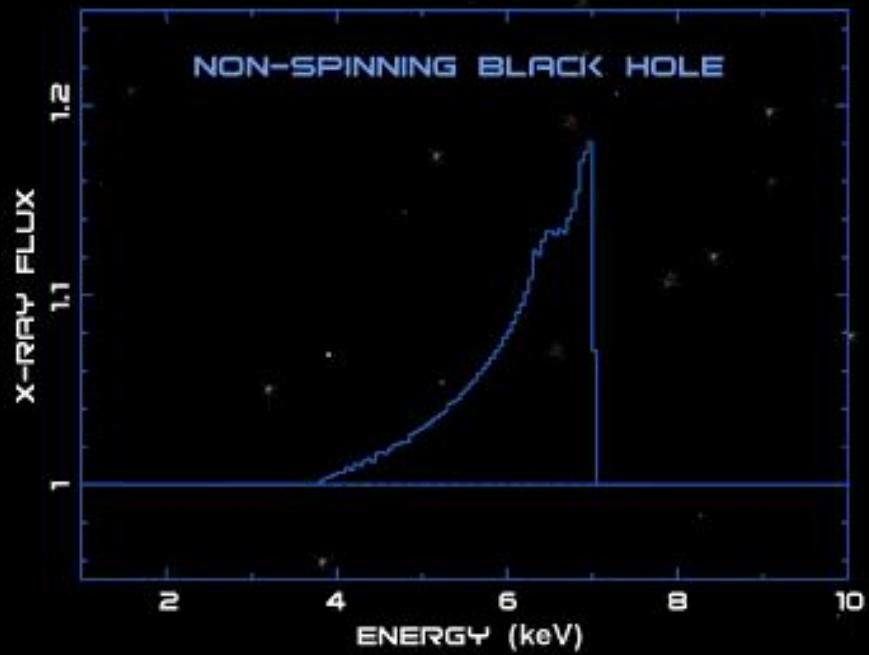
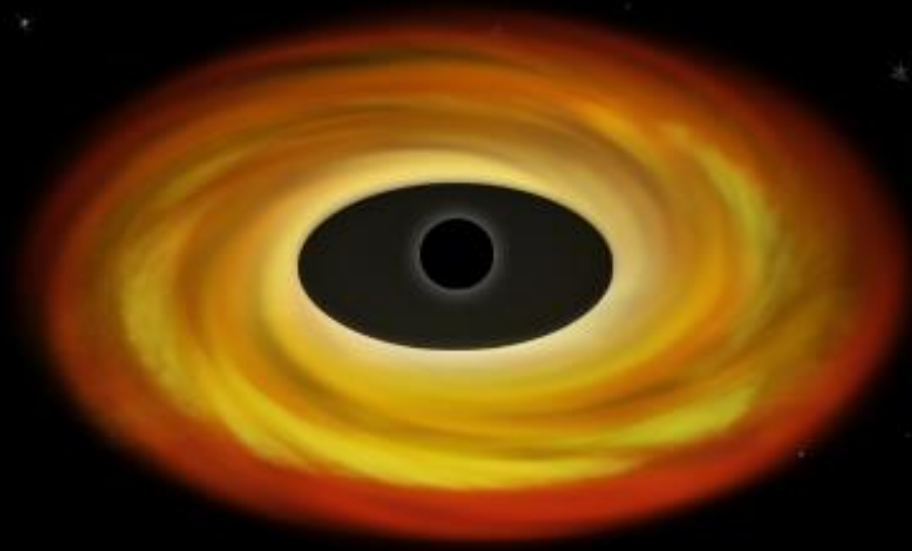
$$= 3.2 \times 10^4 \left(\frac{M}{M_{\odot}} \right) L_{\odot}$$

Soft X-ray Transient



Esin et al. (1997)





Accreting Compact Objects

Low-Mass X-ray Binaries

$$M_c \lesssim 1M_{\odot}$$

Roche-Lobe Overflow

- Main Sequence
- White Dwarf

Wind Fed

- Red Giant
- Slow rotators

High-Mass X-ray Binaries

$$M_c \gtrsim 10M_{\odot}$$

Roche-Lobe Overflow

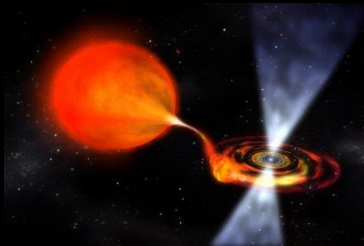
- Supergiant
- $P \lesssim 10s$

Wind Fed

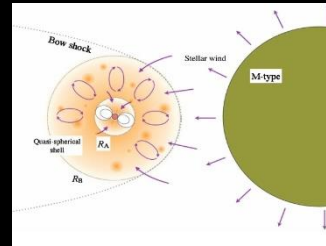
- Supergiant
- Slow rotators
- Direct-wind vs disk-wind

Be X-ray Binaries

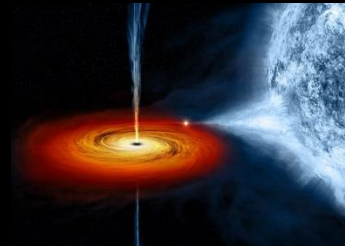
- Be stars
- Equatorial mass outflow
- Regular outbursts



© NASA



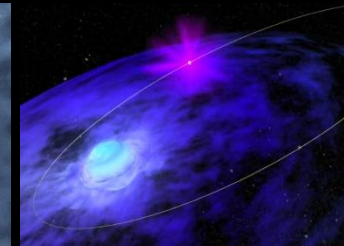
Enoto+2014



© NASA



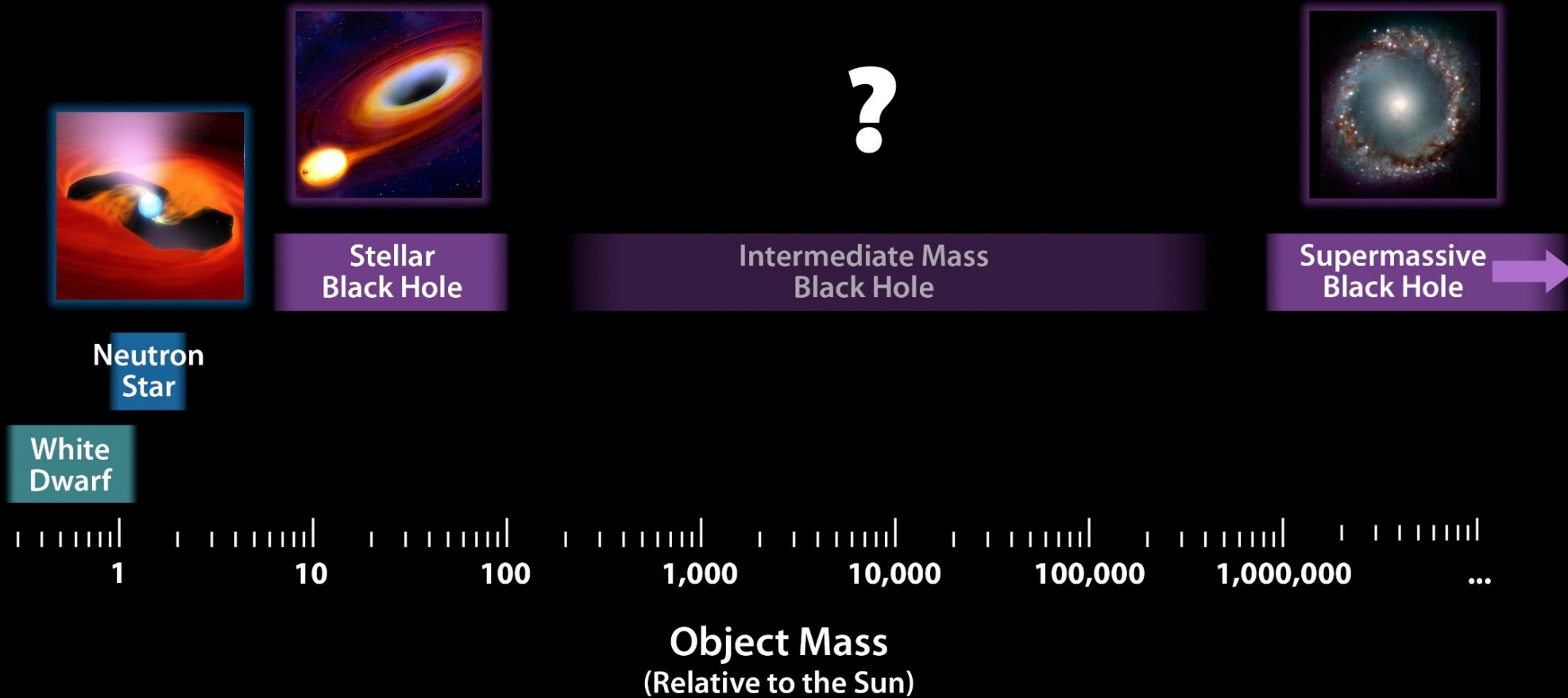
© ESA



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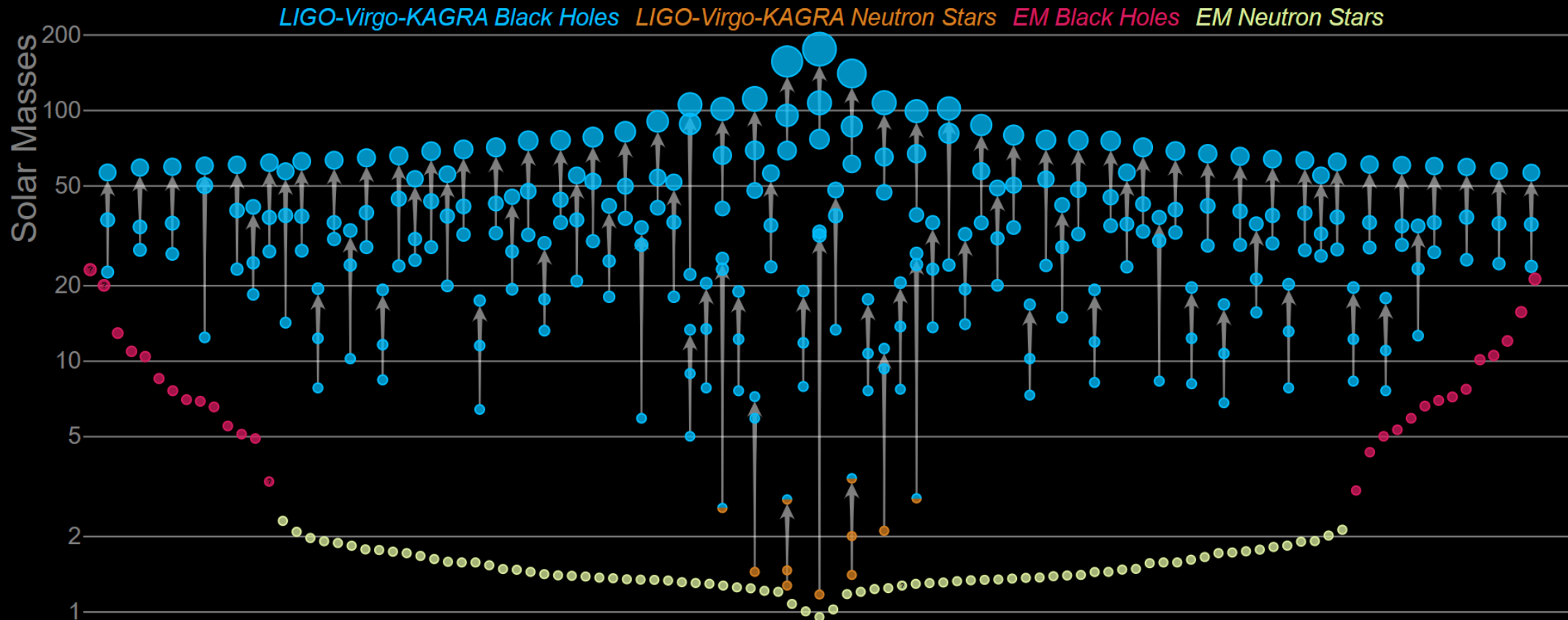
The Mass of Black Holes

Observed Mass Ranges of Compact Objects



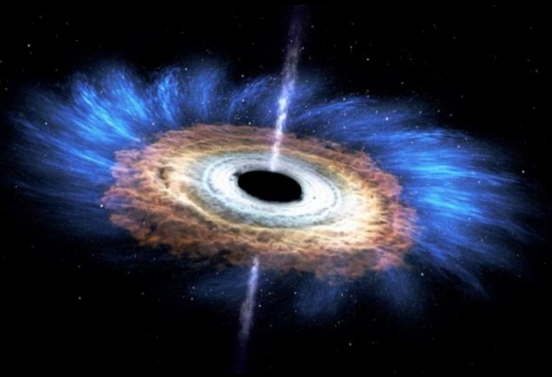
Gravitational Wave: Black Hole Coalescence

Masses in the Stellar Graveyard

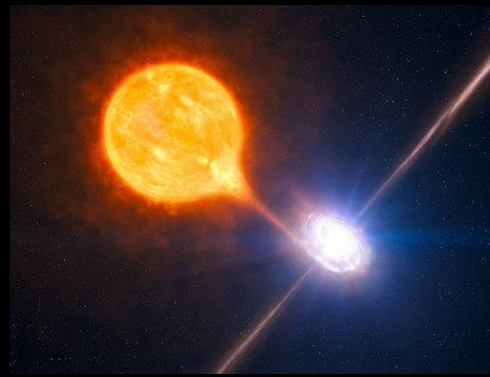


Ultraluminous X-ray Sources (ULX)

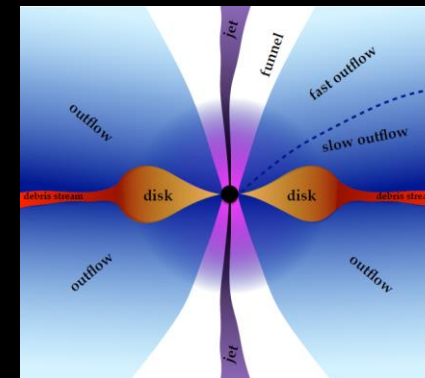
- Extragalactic, non-nuclear objects
 - $L_{0.3-10keV} > 1.3 \times 10^{39} \text{ erg s}^{-1}$ (or $3 \times 10^{39} \text{ erg s}^{-1}$).
 - Intermediate-mass BH with sub-Eddington accretion?
 - Stellar-mass BH with strong beaming (microquasar)?
 - Stellar-mass BH with super-Eddington accretion and mild beaming?



© NASA



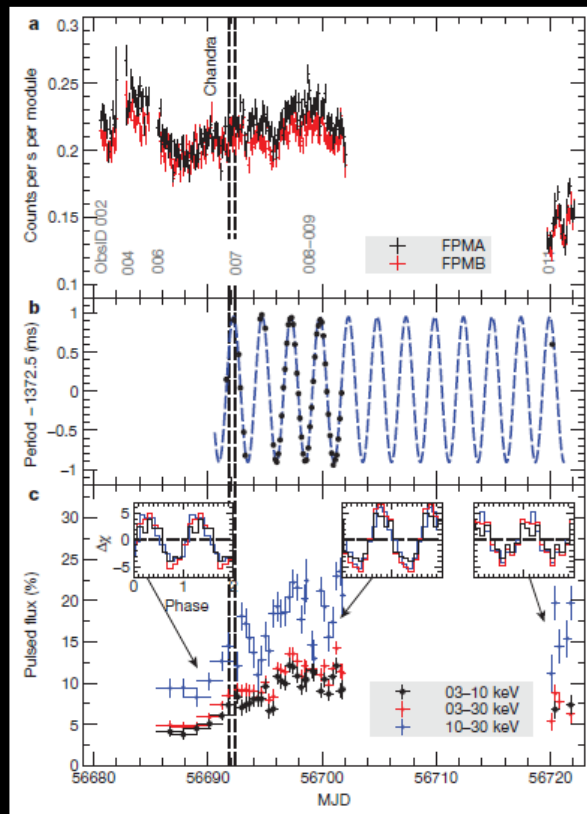
© ESO



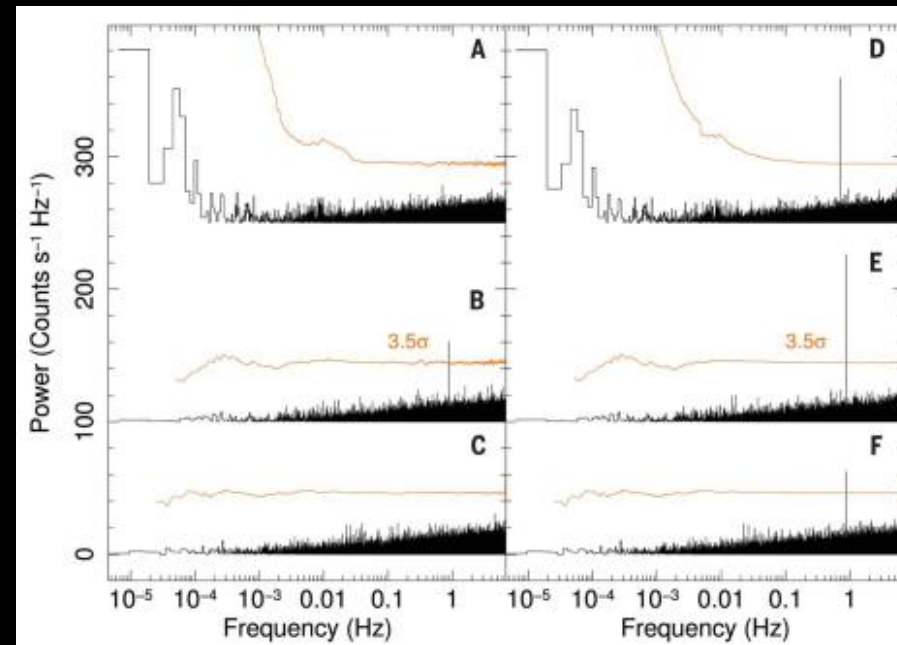
Dai et al. (2017)

Neutron Stars in ULXs!

- Since 2014, astronomers found a few ULXs are powered by neutron stars!



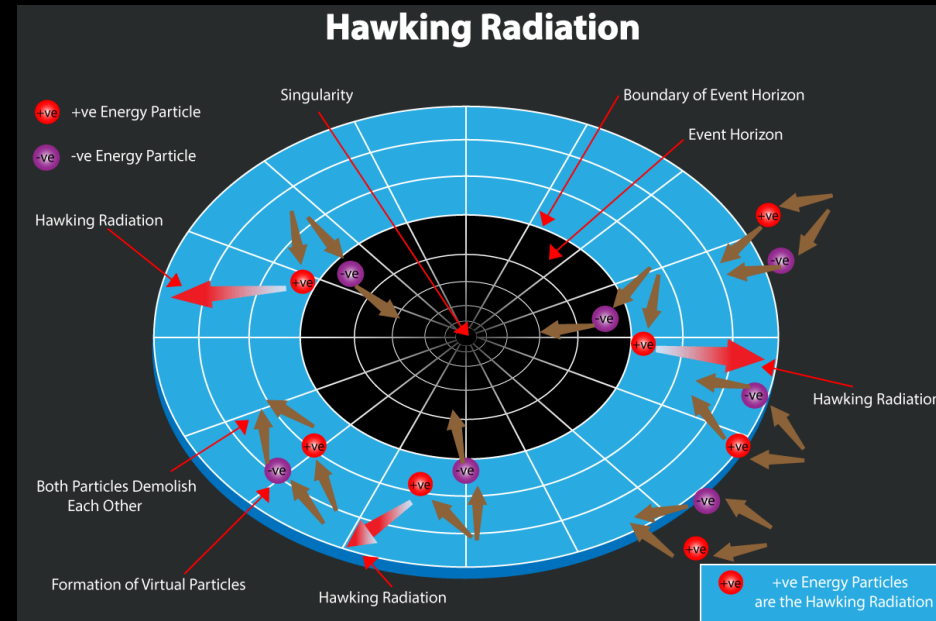
M82 X-2, Bachetti+2014



NGC 5907 ULX-1, Israel+2017

Isolated Black Holes: Hawking Radiation?

- An isolated black hole without accretion cannot be detected with current instruments.
- It may have Hawking radiation, but is too weak to be detected.
- This could be keys to determine Primordial black holes – which are formed in the big bang.



Take home message

- Compact objects are natural laboratories to test physics in extreme conditions.
- These objects are mainly observed in radio (isolated neutron stars) and X-rays (binary systems).
 - Active galactic nuclei could be observed in optical band
- They are powered by rotational energy (pulsars), magnetic energy (magnetars), gravitational potential energy (accreting compact objects)
 - A few extremely faint objects are powered by remnant heat.
- They have fruitful information hidden behind their timing and spectral phenomena in multi-messenger astronomical era.
 - Combination of observational and theoretical works are needed to explore their nature.