國立清華大學 NATIONAL TSING HUA UNIVERSITY

NCTS

MOST 科技部 Ministry of Science and Technology

NARLabs 國家實驗研究院 國家高速網路與計算中心 National Center for High-performance Computing

Radiative process

(Photon/neutrino) radiation transport

Kuo-Chuan Pan (NTHU)

NCTS-TCA Summer Student Program 2023

Outline

- Introduction
- The Boltzmann equation for radiation transport
- Numerical methods for radiative transfer
- (Application: Core-Collapse Supernova)

Introduction

Introduction

Observation

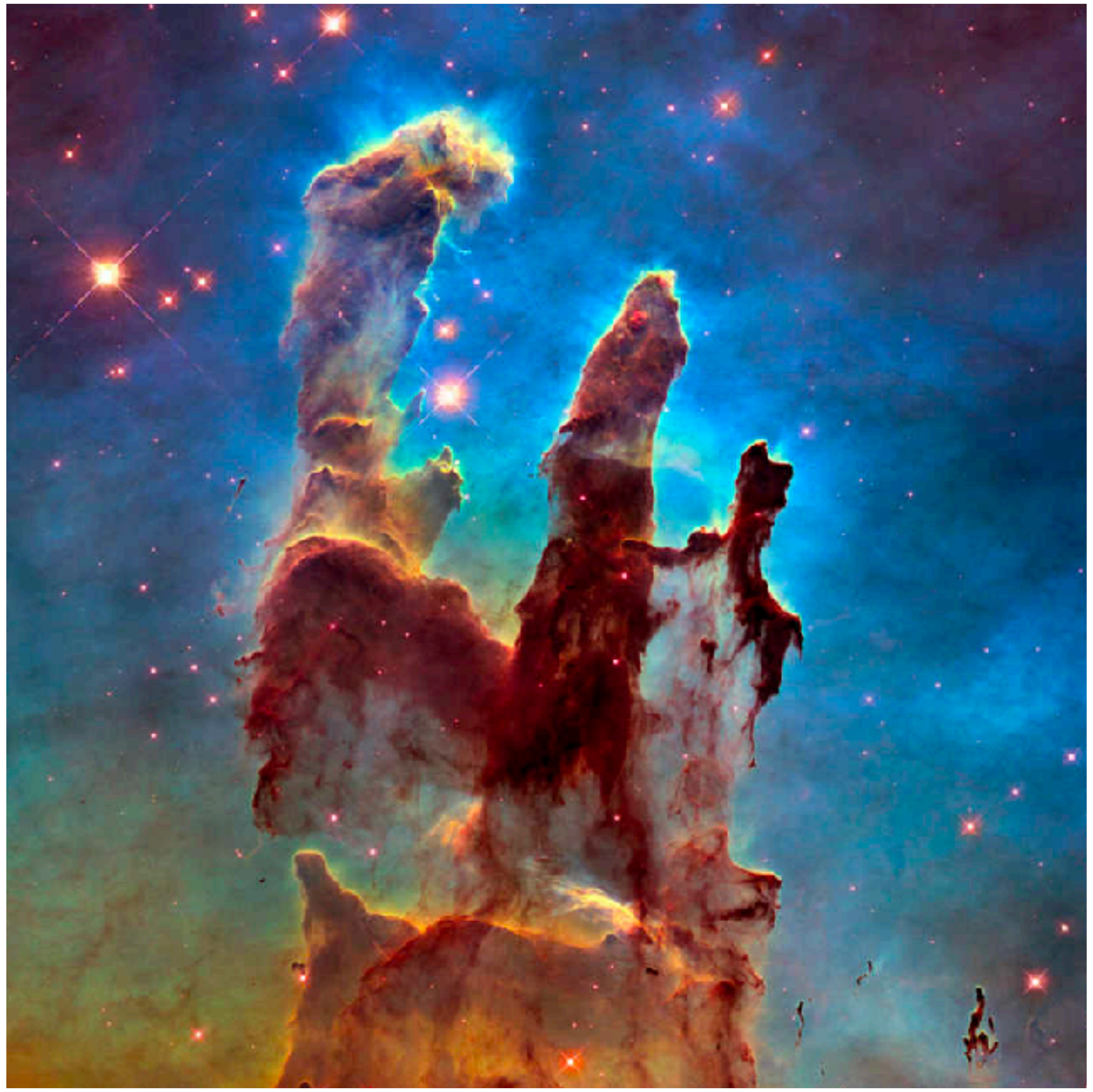


Image credit: NASA

Simulation

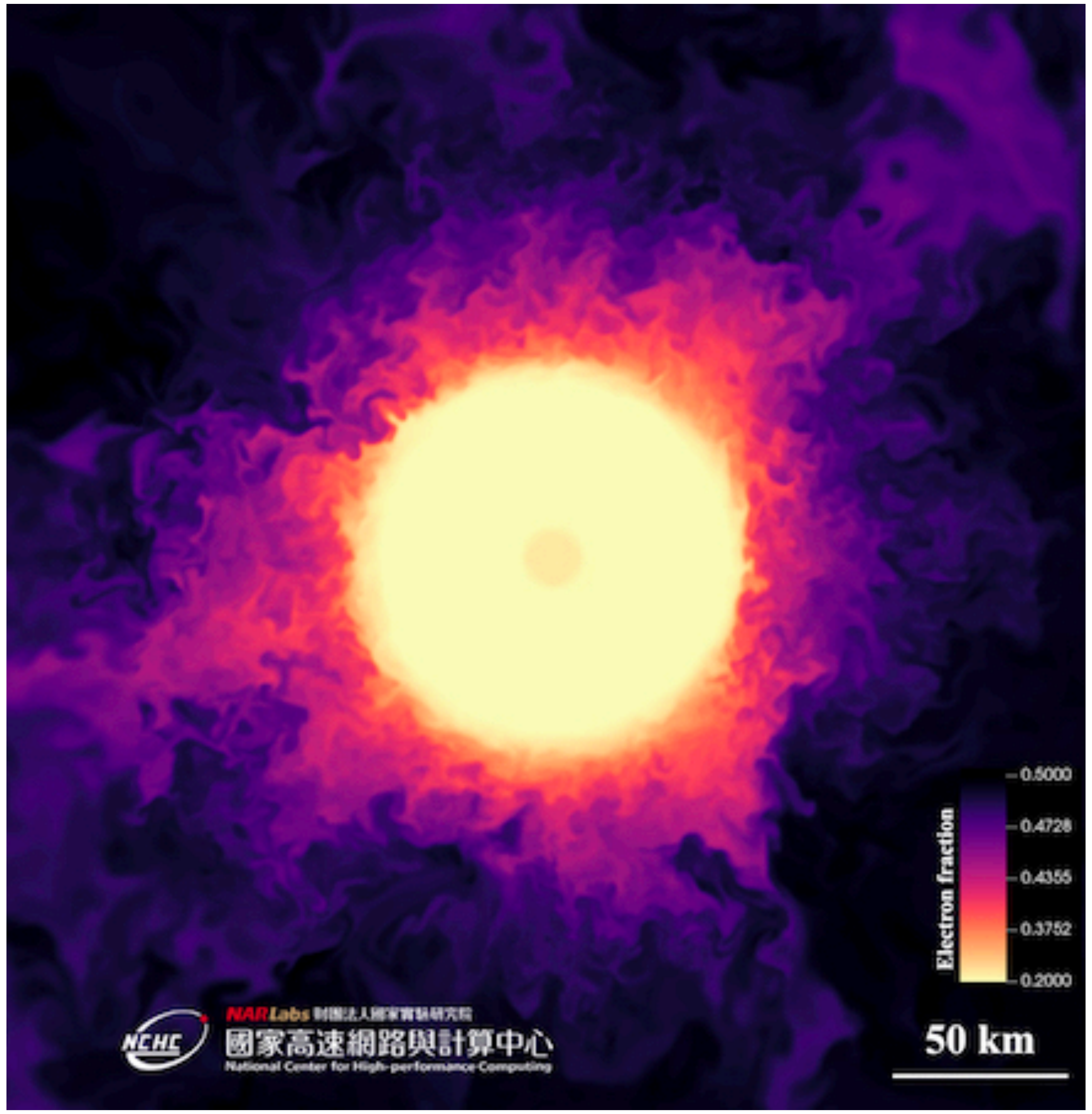


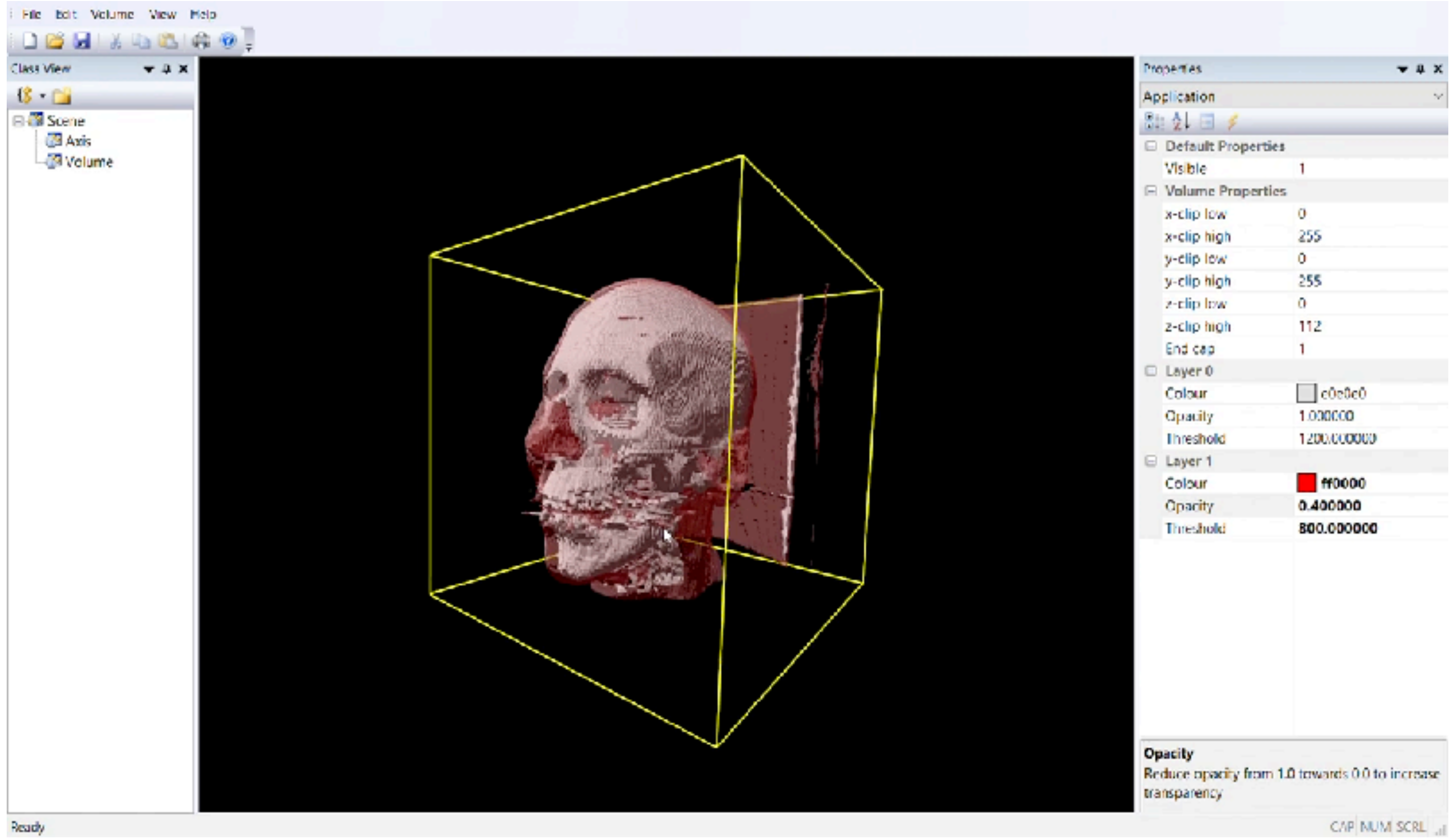
Image credit: K.-C. Pan

Introduction (conti.)

- Given an astrophysical system, how does it looks like?
- Given an observed astronomical object (i.e. an image or spectrum), what is the nature of the physical system?
- Radiative process link astrophysical systems with astronomical observables
- Light curves & spectra
- Chemistry, atomic/molecular lines, neutrino interactions, …etc.
- Radiation feedback
- Radiation fields: EM waves (photons), neutrinos, … (Multi-messengers)

Daily life examples

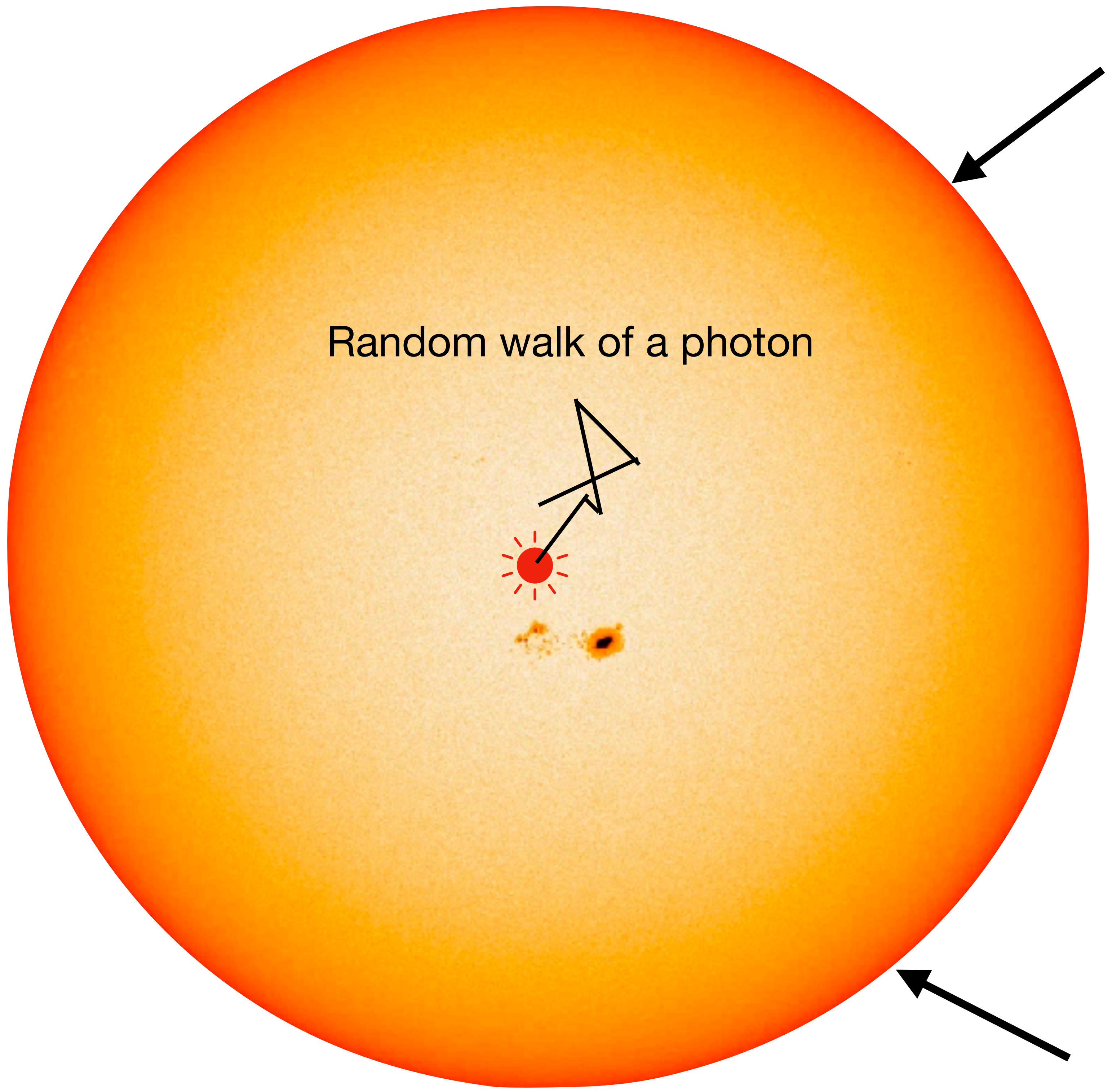
Video credit: https://www.youtube.com/watch?v=CIMFsY_QKBM

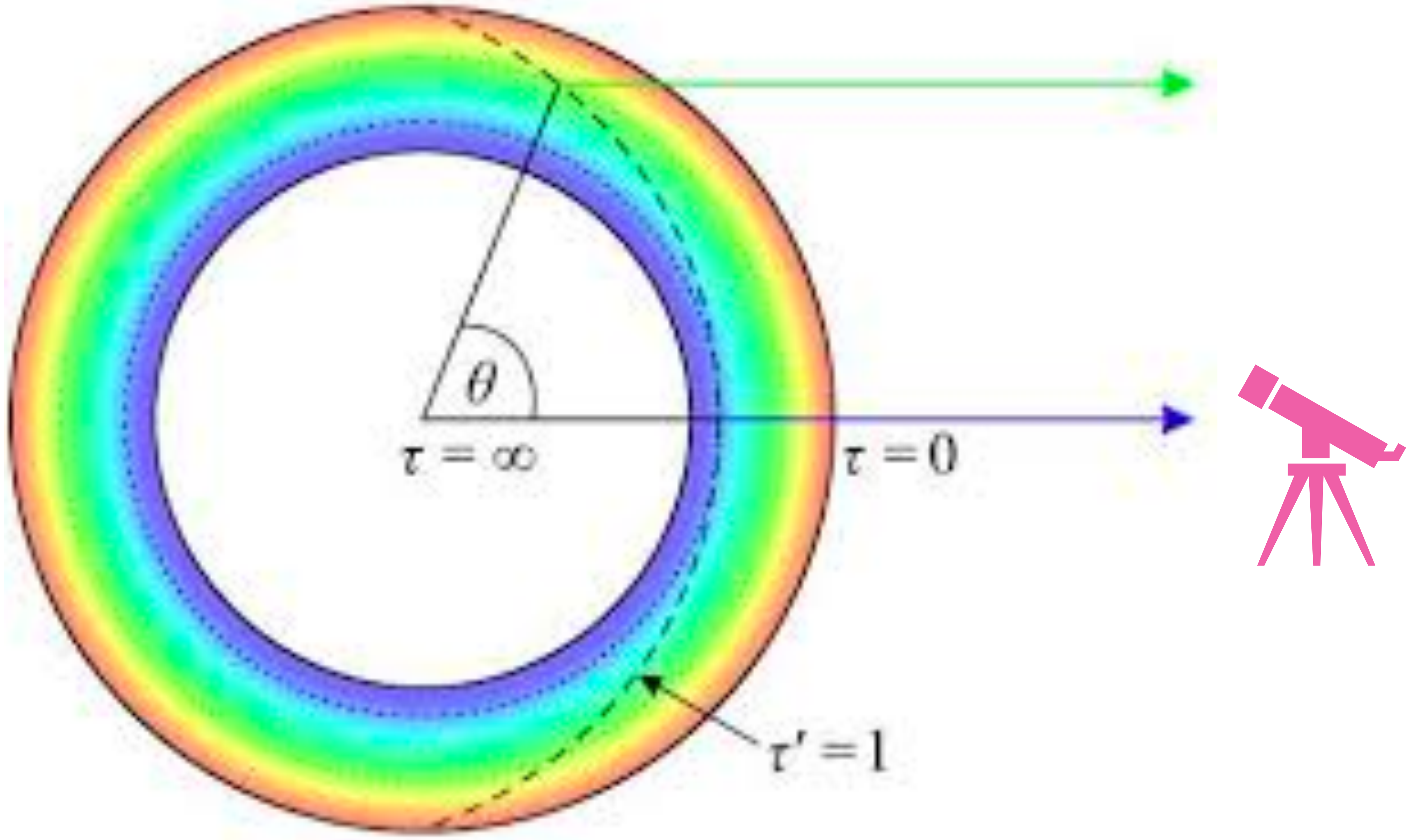


Video credit: https://www.youtube.com/watch?v=9dPlkHiJ6A4

Limb Darkening

Photosphere

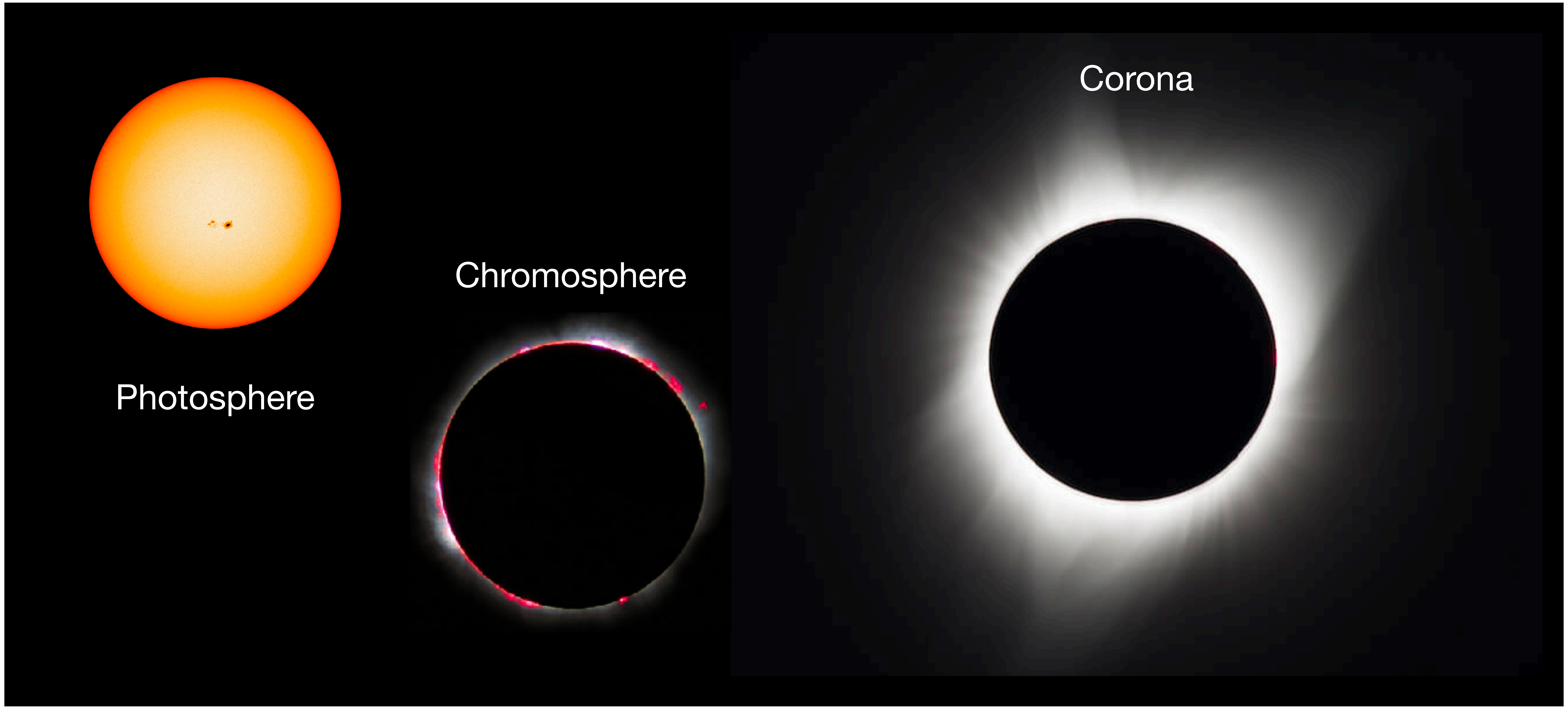


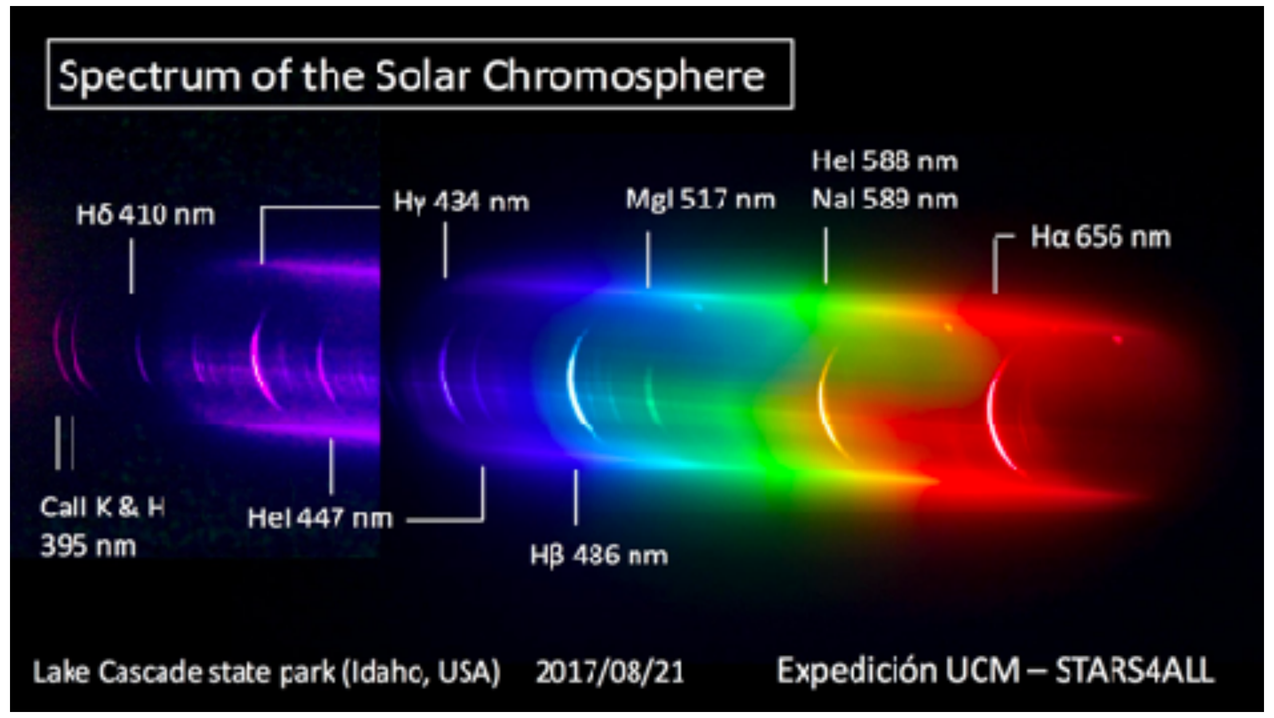


Last scattering surface

Image credit: NASA

Sun’s “Surface”s





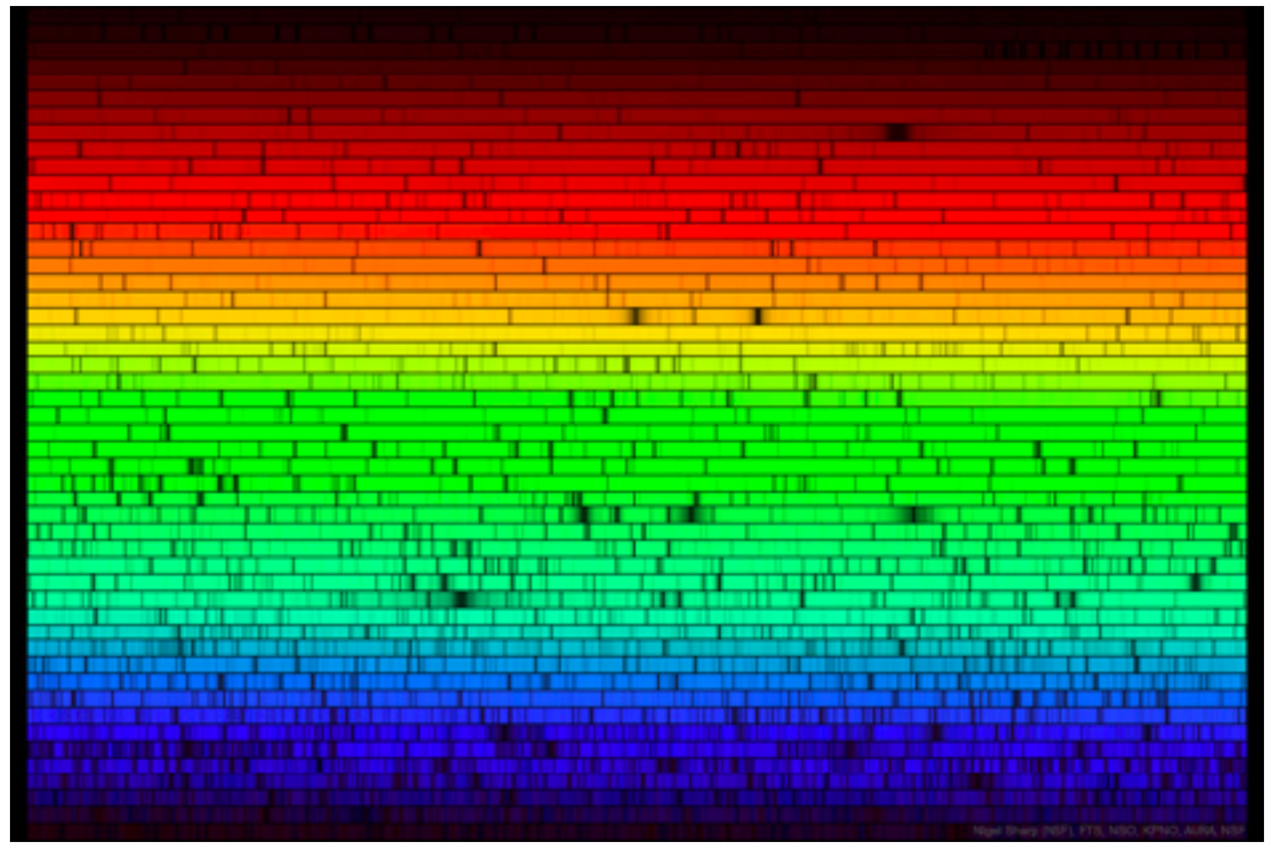


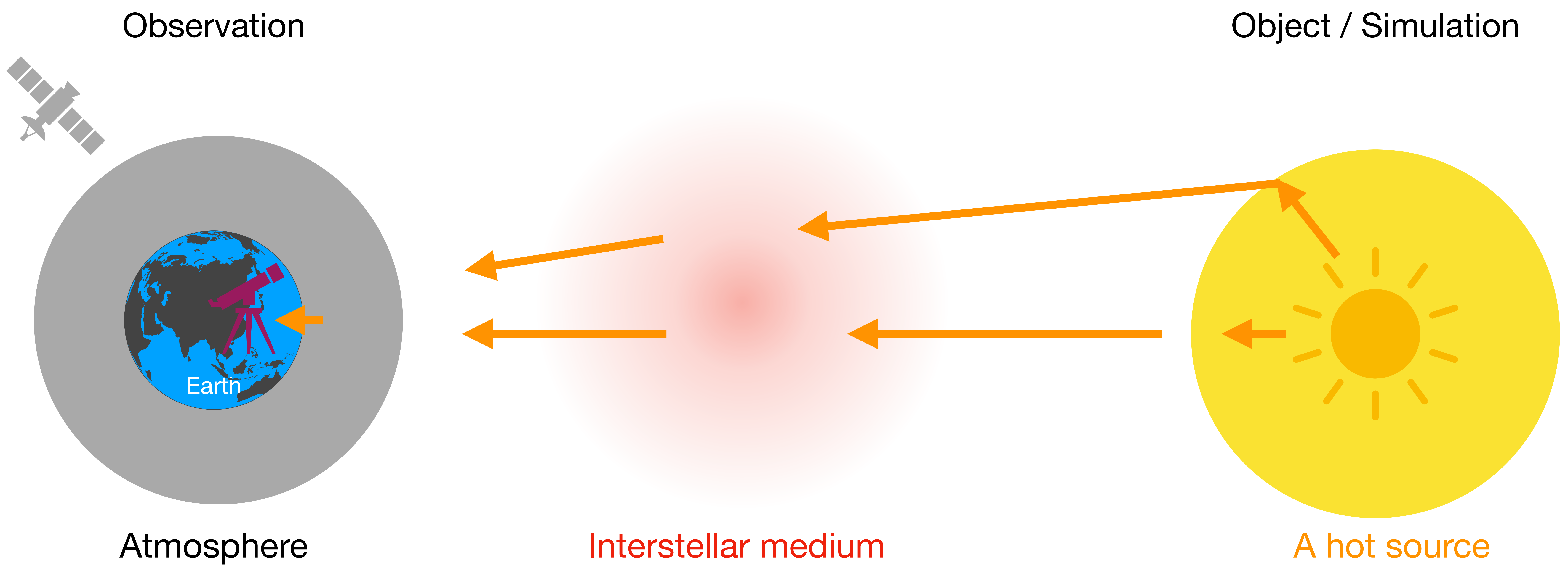
Image credit: NASA

Image credit: Luc Viatour

Image credit: NASA

Image credit: Nigel Sharp

Introduction



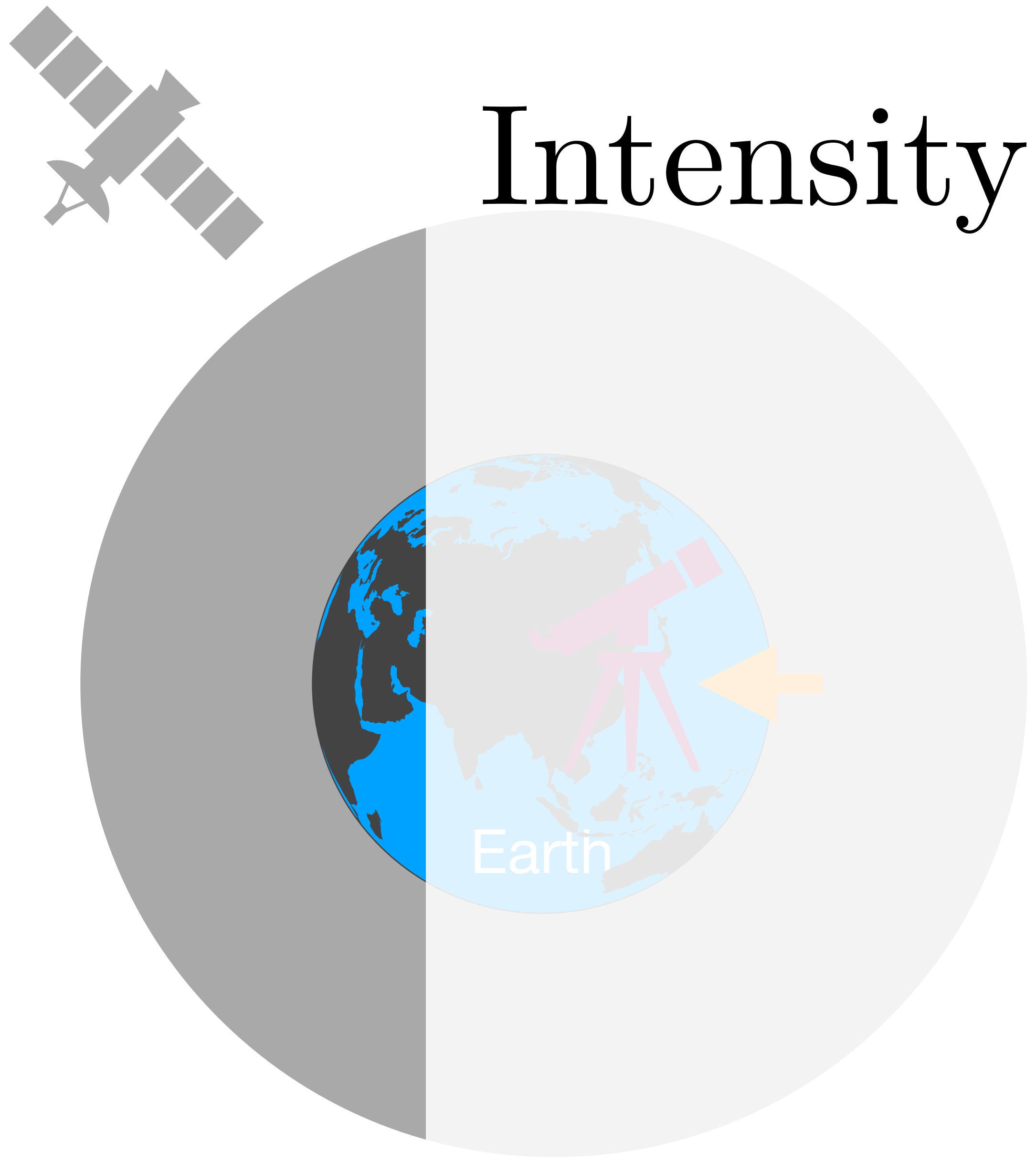
Recall: Hi. Hirashita's Talk

Introduction

Observation

Object / Simulation

Intensity (or brightness) I_ν

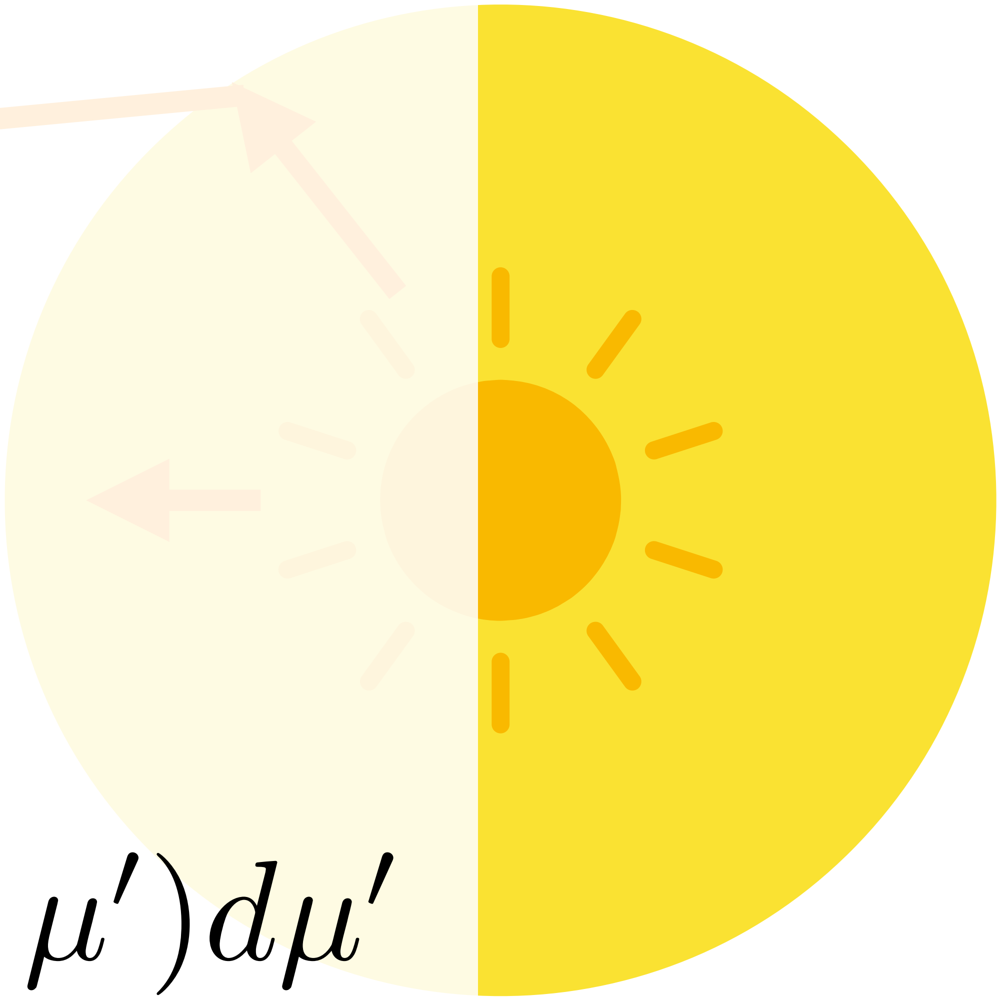


Emission j_ν

Absorption α_ν

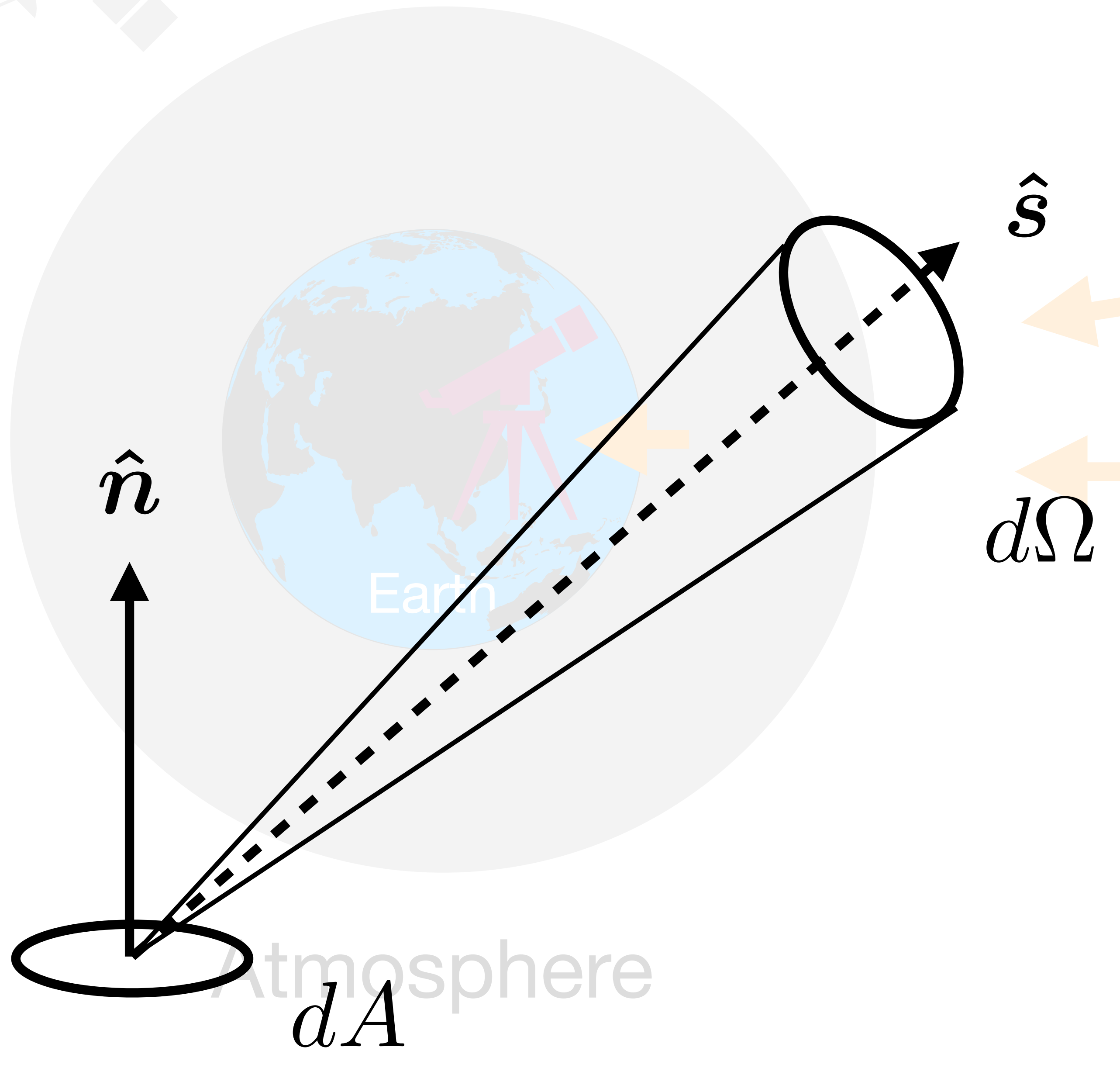
Scattering, $j_\nu = \sigma_\nu \int_{-1}^{1} I'_\nu(\epsilon', \mu') d\mu'$

Interstellar medium



Introduction

Intensity (or brightness) I_ν $\quad [\mathrm{erg\ s^{-1}\ cm^{-2}\ ster^{-1}\ Hz^{-1}}]$



Emission j_ν $\quad [\mathrm{erg\ s^{-1}\ cm^{-3}\ ster^{-1}\ Hz^{-1}}]$

Absorption α_ν $\quad [\mathrm{cm^{-1}}]$

Scattering, $j_\nu = \sigma_\nu \int_{-1}^{1} I'_\nu(\epsilon', \mu') d\mu'$

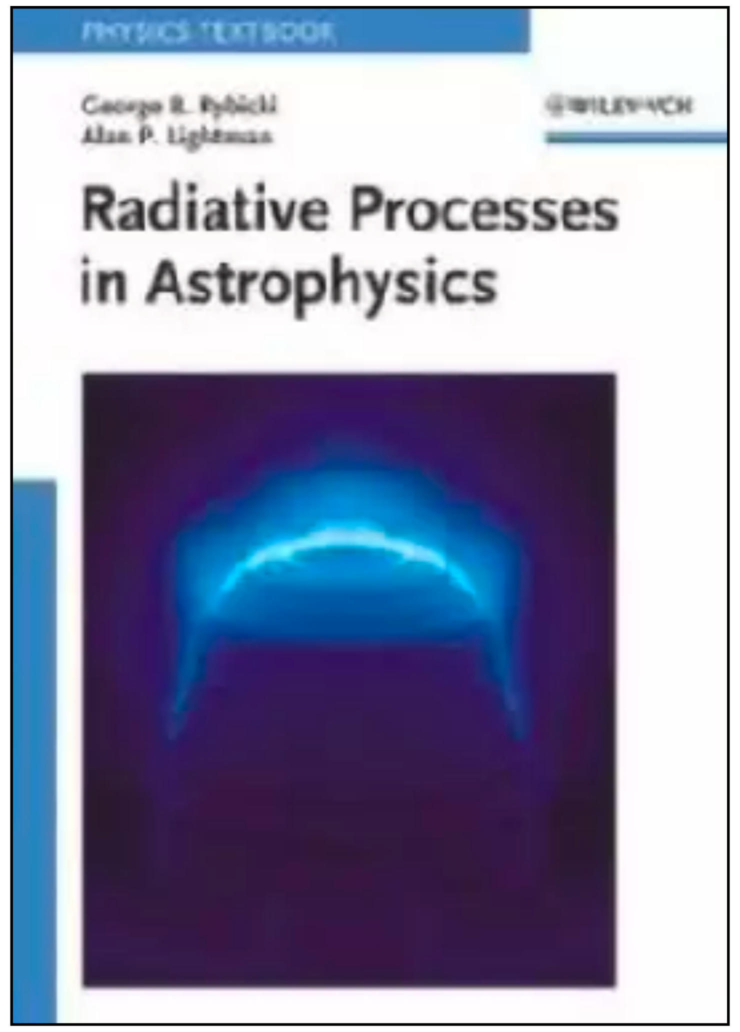
$$dE_\nu = I_\nu(\boldsymbol{x}, \hat{\boldsymbol{s}}, \nu, t)\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{s}} dA d\Omega d\nu dt$$

σ_ν is the absorption coefficient of the scattering process

The Transport Equations

Suggested textbooks

- “Radiative Processes in Astrophysics”, Rybicki & Lightman
- “The Physics of Astrophysics”, Frank Shu



Review: Basic Radiative Transfer Equation

assume no scattering process

$$\frac{dI_\nu}{ds} = \underbrace{-\alpha_\nu I_\nu}_{\text{Absorption}} + \underbrace{j_\nu}_{\text{Emission}}$$

Recall H. Hirashita's talk

Or,

$$\frac{dI_\nu}{d\tau_\nu} = -\left(I_\nu - S_\nu\right)$$

Optical depth, $d\tau_\nu = \alpha_\nu ds$

Source function, $S_\nu = \frac{j_\nu}{\alpha_\nu}$

Opacity, $\kappa_\nu = \frac{\alpha_\nu}{\rho}$ $[\mathrm{cm}^2\ \mathrm{g}^{-1}]$

- Optically thick: $\tau_\nu > 1$
- Optically thin: $\tau_\nu < 1$

Special cases: emission or absorption only

Scattering

$$\text{Scattering } \sigma_\nu \int_{-1}^{1} I'_\nu(\epsilon', \mu') d\mu'$$

- Coherent, isotropic scattering

$$\frac{dI_\nu}{ds} = -\left(\alpha_\nu + \sigma_\nu\right)\left(I_\nu - S_\nu\right)$$

$$\left(\begin{array}{l} \text{Scattering coefficient, } \sigma_\nu \\ \text{Mean intensity, } J_\nu \end{array} \right.$$

$$S_\nu = \frac{\alpha_\nu j_\nu + \sigma_\nu J_\nu}{\alpha_\nu + \sigma_\nu}$$

Average of two source functions, weighted by their respective absorption coefficients

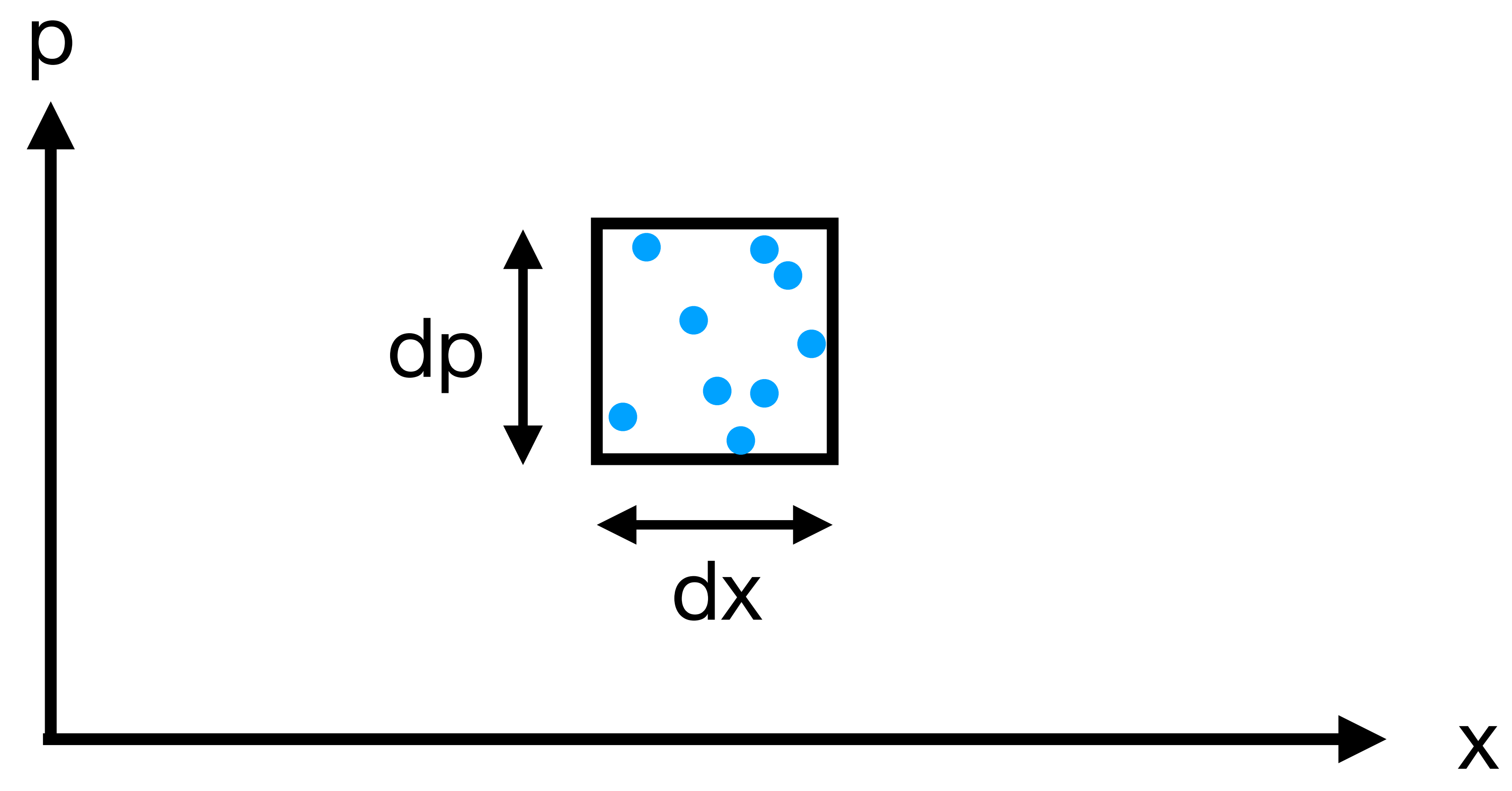
Derived from Boltzmann equation

- Consider a cavity containing a gas of particles. The mean number of particles in this cavity is

$$N = \int f(x, p, t) d^3x d^3p,$$

where f is the distribution function. (x,p) is the phase spaces of momentum and position coordinates.

The Boltzmann equation



- Particles are subject to an external force field "F"
- The Boltzmann equation

$$f(\boldsymbol{x} + \boldsymbol{u}dt, m\boldsymbol{u} + \boldsymbol{F}dt, t + dt) - f(\boldsymbol{x}, \boldsymbol{p}, t) = [\Delta f]_{\text{coll.}}$$

The Boltzmann equation

$$f(\boldsymbol{x} + \boldsymbol{u}dt, m\boldsymbol{u} + \boldsymbol{F}dt, t + dt) - f(\boldsymbol{x}, \boldsymbol{p}, t) = [\Delta f]_{\mathrm{coll.}}$$

Recall H.-Y. Pu's talk

$$\boxed{\frac{df}{dt} = \left[\frac{\partial f}{\partial t}\right]_{\mathrm{coll.}}}$$

or

$$\boxed{\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial u_i} = \left[\frac{\partial f}{\partial t}\right]_{\mathrm{coll.}}}$$

- The evolution of the distribution function in the six dimensional space

Number density $$n(x,t) = \int f(x,p,t)dp$$

Mass density $$\rho(x,t) = \int mf(x,p,t)dp$$

Bulk velocity $$v(x,t) = \int muf(x,p,t)dp$$

The Boltzmann equation

- When collisions are "elastic" and the density of the medium is low enough for collisions involving more than two particles to be neglected
- Then, in absence of external force, f is obtained from statistical mechanism and is given by the Maxwellian velocity distribution

$$f(x, \boldsymbol{u}, t) d\boldsymbol{u} = n(x,t) \left[\frac{m}{2\pi k T(x,t)} \right]^{\frac{3}{2}} \exp \left[- \frac{m(u-v)^2}{2kT(x,t)} \right]$$

* Using the same concept, we could derive the hydrodynamics equations as well

The Boltzmann equation for photons

$$f(\boldsymbol{x}+\boldsymbol{u}dt, m\boldsymbol{u}+\boldsymbol{F}dt, t+dt) - f(\boldsymbol{x},\boldsymbol{p},t) = [\Delta f]_{\text{coll.}}$$

Recall H.-Y. Pu's talk

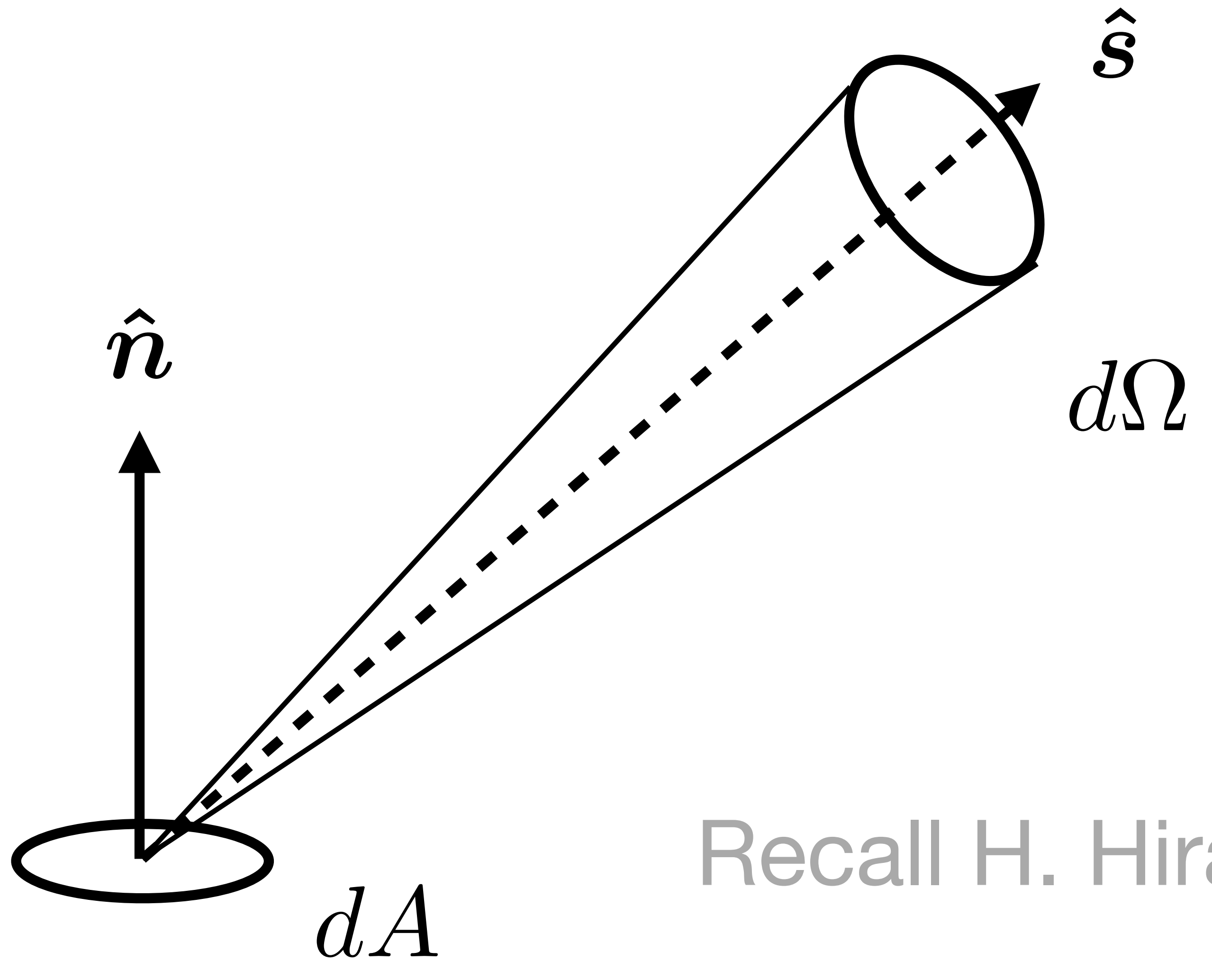
$$\boxed{\frac{df}{dt} = \left[\frac{\partial f}{\partial t}\right]_{\text{coll.}}} \quad \text{or} \quad \boxed{\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial u_i} = \left[\frac{\partial f}{\partial t}\right]_{\text{coll.}}}$$

- Photon transport: $f = f_\gamma$

$$dE_\nu = h\nu f_\gamma(\boldsymbol{x},\boldsymbol{p},t)d\boldsymbol{x}d\boldsymbol{p}$$

$$dE_\nu = I_\nu(\boldsymbol{x},\hat{\boldsymbol{s}},\nu,t)\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{s}}dAd\Omega d\nu dt$$

$$I_\nu = (h\nu/c)(h^2\nu)f_\gamma = \frac{h^4\nu^3}{c^2}f_\gamma = C_1 f_\gamma$$



Recall H. Hirashita's talk

Photon Transport Equation

$$\frac{\partial f}{\partial t} + u_i \frac{\partial f}{\partial x_i} + F_i \frac{\partial f}{\partial u_i} = \left[\frac{\partial f}{\partial t}\right]_{\mathrm{coll.}}$$

- Photon transport: $f = f_\gamma$

$$\frac{1}{C_1}\left[\frac{\partial I_\nu}{\partial t} + c(\hat{\boldsymbol{s}} \cdot \nabla) I_\nu\right] = \frac{1}{C_1}\left[\frac{\partial f}{\partial t}\right]_{\mathrm{coll.}}$$

$\boldsymbol{F} = 0$ for Newtonian photons

$$\boxed{\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{\boldsymbol{s}} \cdot \nabla I_\nu = -\epsilon_\nu I_\nu + j_\nu + [\text{Other scattering terms}]}$$

Extinction, $\epsilon_\nu = \sigma_\nu + \alpha_\nu$

Moments of the Boltzmann equation for Photons

- The moments of the Boltzmann equation define the dynamical equations for the radiation field.

0 th moment: mean Intensity

$$J_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu d\Omega,$$

1st moment: radiation flux

$$H_\nu^i = \frac{1}{4\pi}\int_{4\pi} I_\nu s_i d\Omega,$$

2nd moment: tensor

$$K_\nu^{ij} = \frac{1}{4\pi}\int_{4\pi} I_\nu s_i s_j d\Omega,$$

$$\longrightarrow$$

$$\frac{1}{c}\frac{\partial J_\nu}{\partial t} + \nabla\cdot\boldsymbol{H}_\nu + \alpha_\nu\rho(J_\nu - B_\nu) = 0$$

Radiation energy equation

$$\frac{1}{c}\frac{\partial H_\nu^i}{\partial t} + \sum_j \frac{K_\nu^{ij}}{\partial x_j} + \epsilon_\nu\rho H_\nu^i = 0$$

Radiation momentum equation

Additional closure relations are necessary for “K^{ij}”

Example: Optically thick limit

- e.g. the interior of a star, mean free path is much less than the radius of the star —> local thermodynamics equilibrium (LTE)

$$\boxed{I_\nu \sim B_\nu(T) \text{ and } S_\nu = B_\nu(T)}$$

$$\left(\begin{array}{l} \cancelto{0}{\dfrac{1}{c}\dfrac{\partial J_\nu}{\partial t}} + \nabla \cdot \boldsymbol{H}_\nu + \alpha_\nu \rho (J_\nu - B_\nu) = 0 \\[2ex] \cancelto{0}{\dfrac{1}{c}\dfrac{\partial H^i_\nu}{\partial t}} + \boxed{\displaystyle\sum_j \frac{K^{ij}_\nu}{\partial x_j} + \epsilon_\nu \rho H^i_\nu = 0} \end{array} \right. \longrightarrow$$

Equilibrium (thermal timescale is long)

almost isotropic $K_\nu \sim \frac{1}{3} J_\nu$

$$H_\nu = -\frac{1}{\rho\alpha_\nu} \boldsymbol{\nabla} \cdot \boldsymbol{K_\nu} = -\frac{1}{3\rho\alpha_\nu} \frac{\partial B_\nu}{\partial T} \nabla T$$

$$F_\nu = 4\pi H_\nu \text{ and } F = \int_0^\infty F_\nu d\nu$$

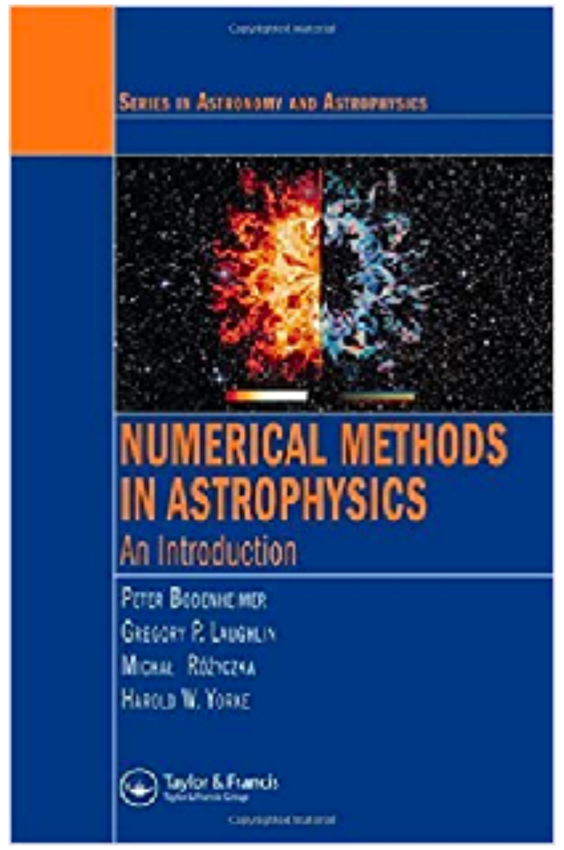
$$F = -\frac{4\pi}{3\rho\alpha_\nu} \nabla T \int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu = \boxed{-\frac{4\pi}{3\rho\alpha_\nu} \nabla (aT^4)}$$

Diffusion approximation

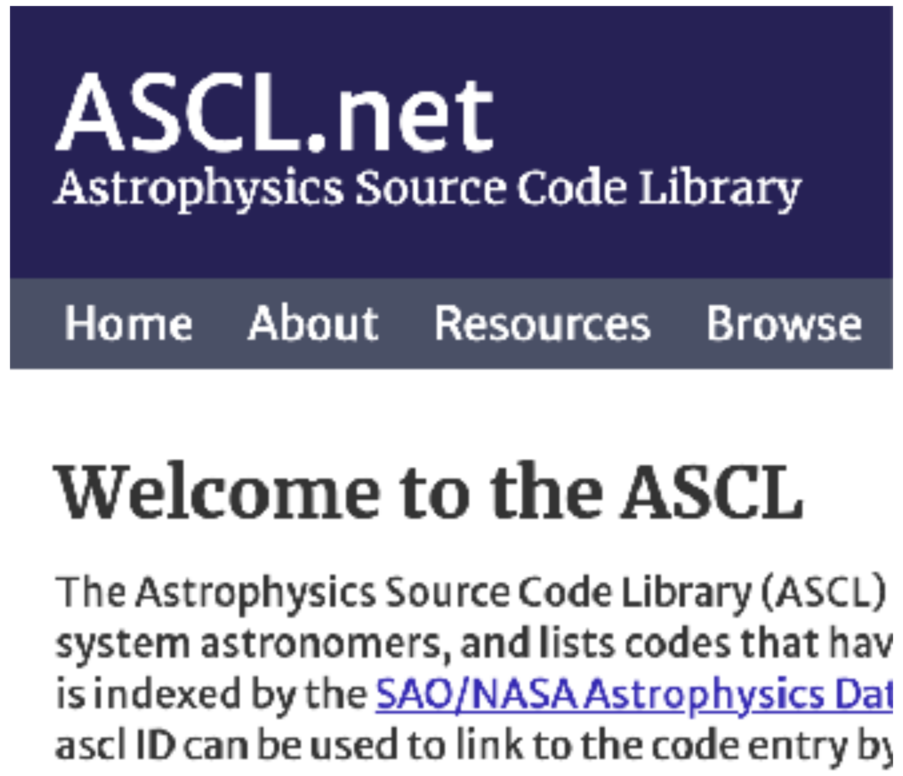
Numerical methods for solving transport equations

Codes and Methods

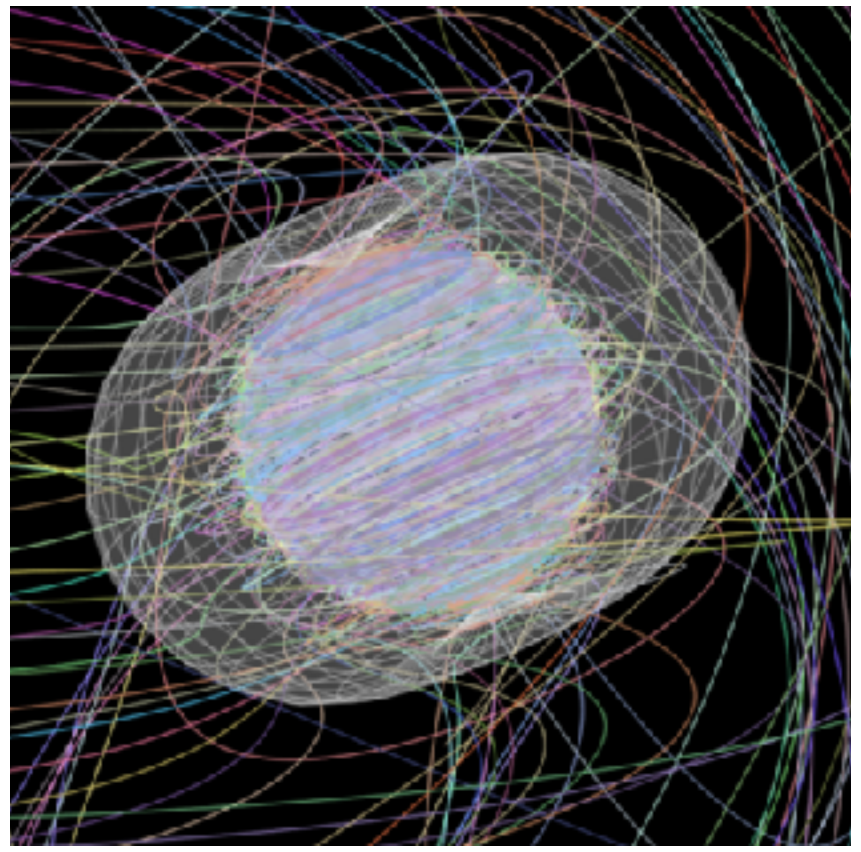
- "Numerical methods in Astrophysics", Bodenheimer et al.
- Astrophysics Source Code Library (http://ascl.net/)
- Odssey.edu (https://odysseyedu.wordpress.com/) by H.-Y. Pu
- More …



Bodenheimer et al.



http://ascl.net



Pu and Yun (2016)

Numerical Challenging

- Can be time-dependent or time independent
- Can be coupled with gas (hydro/mhd) or via post-processing
- (magneto-) hydrodynamics: 4D (t, x, y, z)
- Radiation transport: 7D (t, x, y, z, theta, phi, e/f) —> slow to compute
- If 100 gird points in each dimension —> 10^{12} points per time step (~ 8TB)
- Could have a wide opacity range (from optically thin to thick)
- Additional complexity from multi-dimensional fluids
- Approximations are usually necessary

Numerical approaches - Outline

Non- Transport

- Efficient / Ad-hoc / Poor: polytropic EoS
- Tabulated heating /cooling
- Photon/Neutrino leakage

Rad. Transport

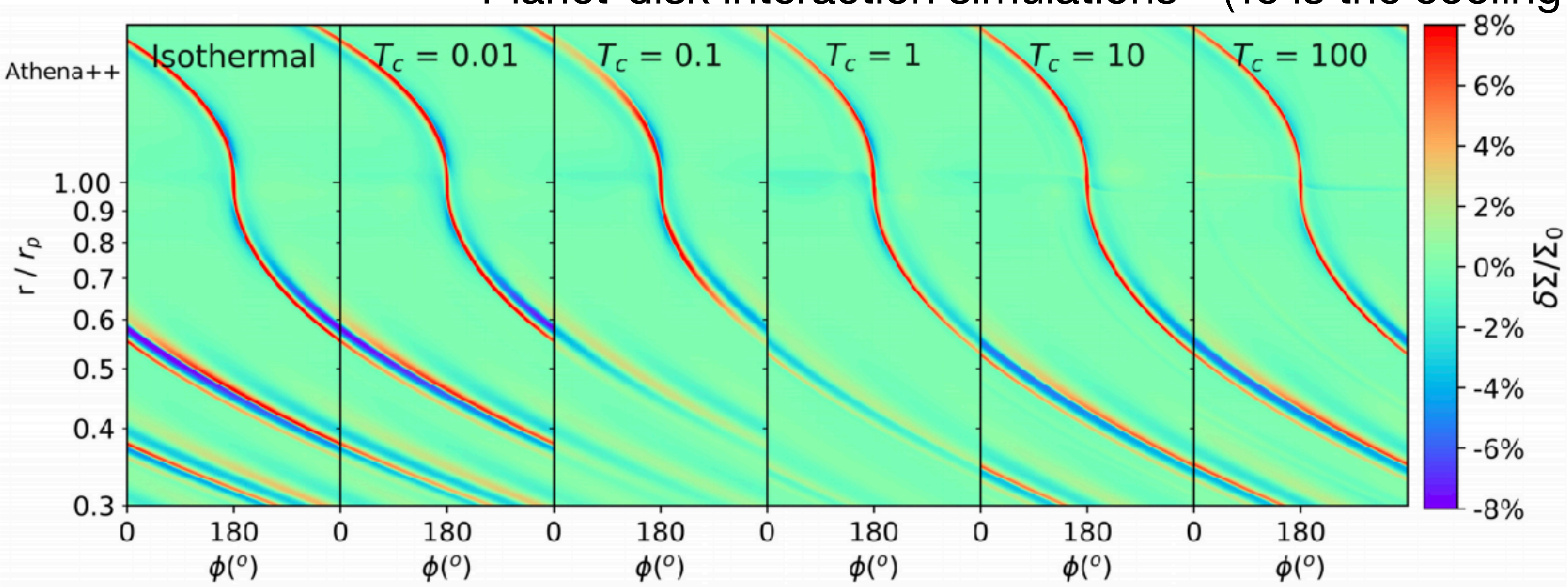
- Flux-limited diffusion
- Ray-tracing
- Moment schemes
- Boltzmann transport

* Each method could have several variants

Non-transport: Isothermal Equation of State(EoS)

- If radiative cooling is so efficient that the isothermal assumption is applicable
- Simply use the isothermal EoS (gamma=1) or polytropic EoS (gamma >~ 1)

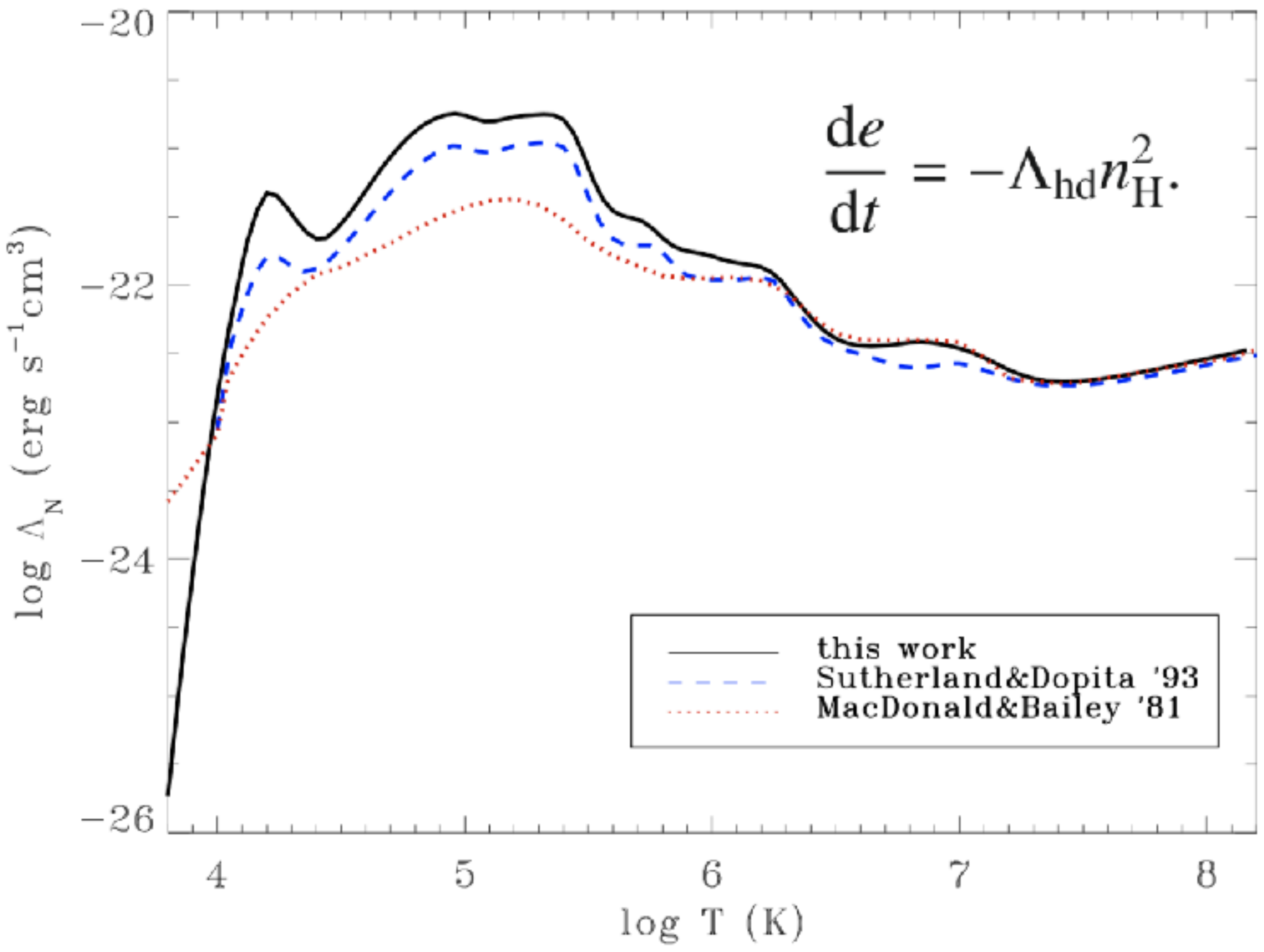
Planet-disk interaction simulations (Tc is the cooling parameter)

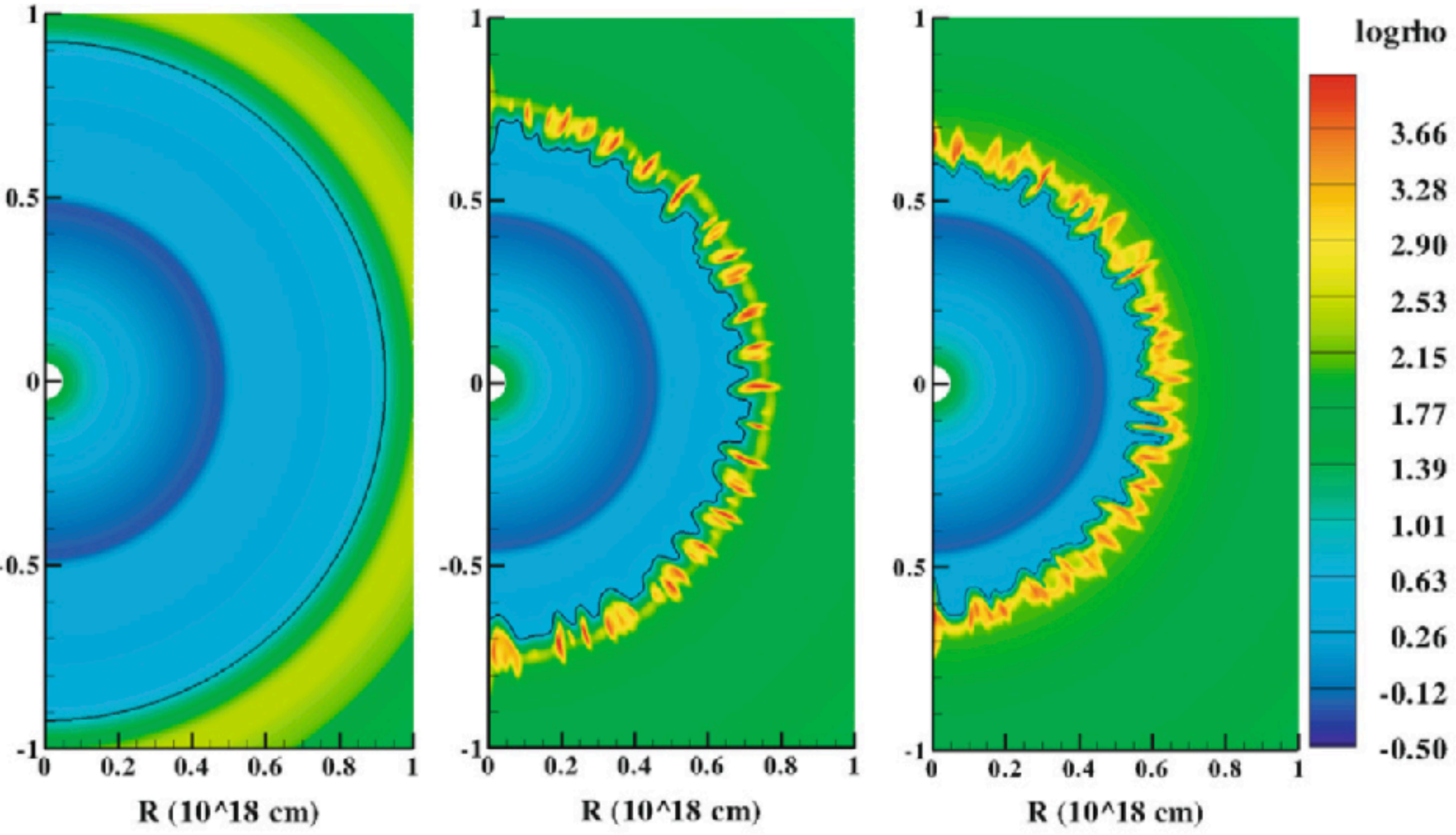


Zhang et al. (2020)

Non-transport: Tabulated heating/cooling

- Add the cooling curve (from a table or a formula) in the energy equation.

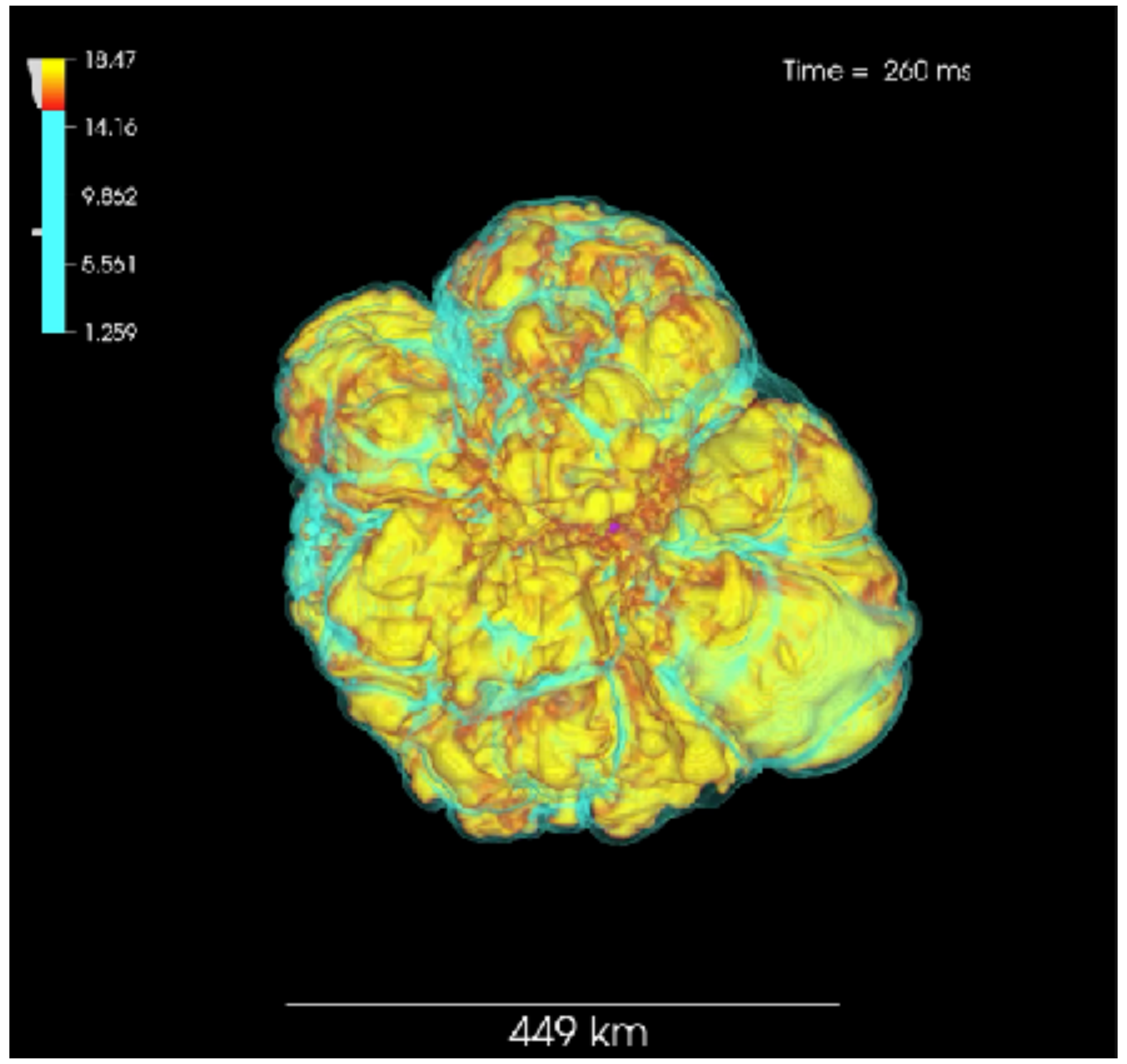




Schure et al. (2009)

Non-transport: Leakage scheme

- The leakage scheme provides approximate energy and number emission/absorption rates based on local thermodynamics and the optical depth.
- The rate of energy emission cab be determined by the interpolation between two limiting regimes
- The optical depth requires a non-local calculation



Couch & O'Connor (2013)

Rad.-transport: FLD

- Radiation-hydrodynamics equations:

$$
\begin{cases}
\dfrac{d\rho}{dt} + \rho\dfrac{\partial v_j}{\partial x_j} = 0 \\[2ex]
\rho\dfrac{dv_i}{dt} = -\dfrac{\partial P_g}{\partial x_i} \boxed{+ \dfrac{1}{c}\epsilon_F \rho F_{\mathrm{rad},i}} \\[2ex]
\rho\dfrac{de}{dt} + (e + P_g)\dfrac{\partial v_j}{\partial x_j} = \boxed{-4\pi\kappa_P \rho B + c\kappa_E \rho u}
\end{cases}
$$

* ignore gravity, magnetic fields, and viscosity

F_{rad} (or F), radiation flux integrated over frequency
B, the Planck function integrated over frequency
u, energy density in the radiation field

$$
\begin{cases}
\kappa_E = \dfrac{1}{u}\displaystyle\int_0^\infty \kappa_\nu u_\nu d\nu, \\[2ex]
\kappa_P = \dfrac{1}{B}\displaystyle\int_0^\infty \kappa_\nu B_\nu(T) d\nu, \\[2ex]
\epsilon_F = \dfrac{1}{F}\displaystyle\int_0^\infty \epsilon_\nu F_\nu d\nu,
\end{cases}
$$

Rad.-transport: FLD (conti.)

- Diffusion Approximation (optically thick limits)
- Frequency integrated (gray) or with different frequency bins (multi-groups)

$$F = -\frac{c}{3\kappa_R \rho}\boldsymbol{\nabla} u, \qquad \text{or} \qquad F = -\frac{c\lambda}{\kappa_R \rho}\boldsymbol{\nabla} u,$$

- In the optical thin limit, flux becomes unphysical large
- We need to adjust lambda in optically thin limit (need a flux limiter)

Rad.-transport: FLD (conti.)

- Flux-limited diffusion
- Define a dimensionless quantity R

$$R = \frac{|\nabla u|}{\kappa_R \rho u},$$

Is the ratio of mean free path to the energy scale height

$$\lambda = \frac{2+R}{6+3R+R^2}$$

$$\left(\begin{array}{l} R- > 0 \text{ optically thick } \lambda = \frac{1}{3} \\ R- > \infty \text{ optically thin } \lambda = \frac{1}{R} \end{array} \right.$$

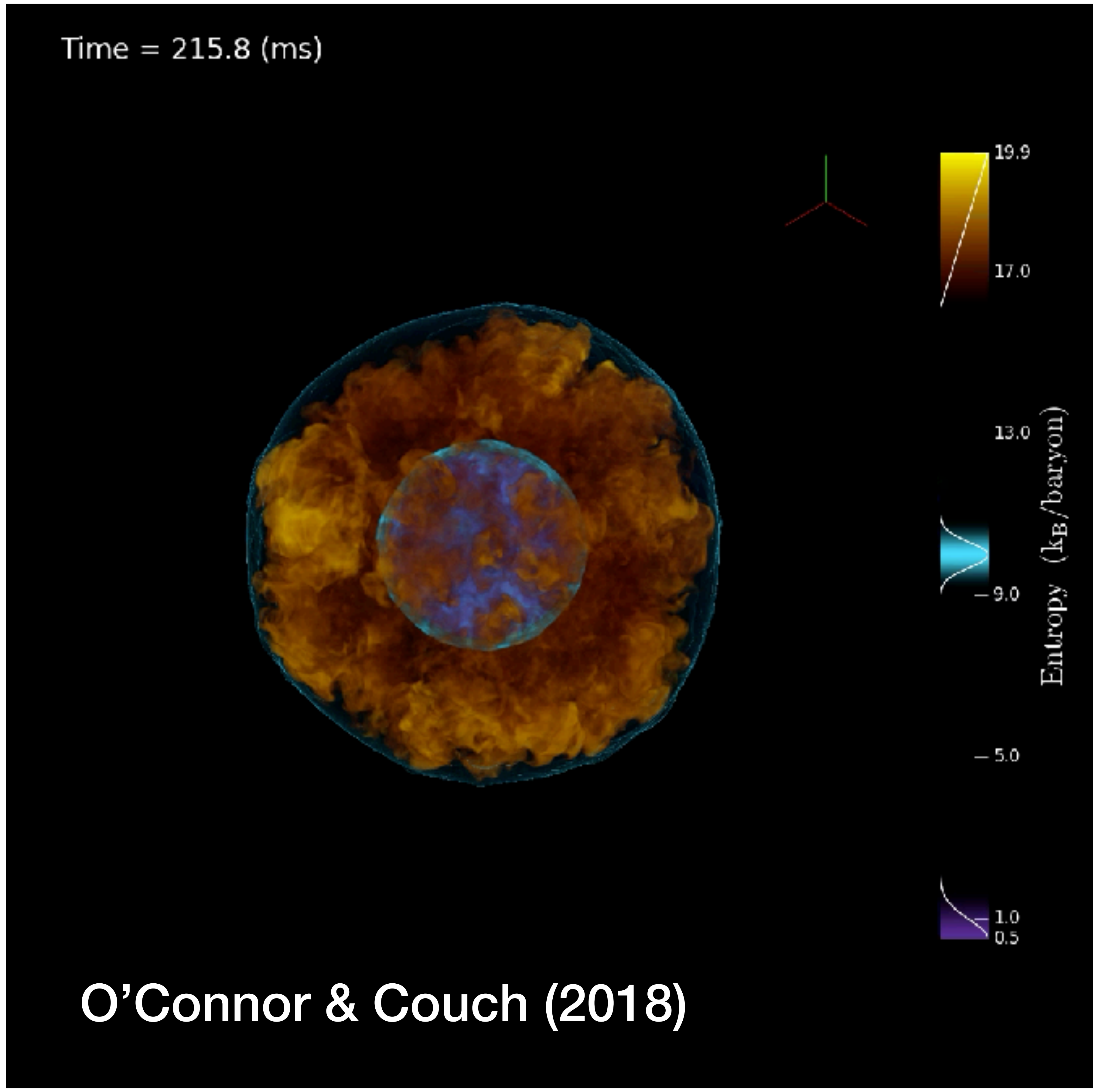
Levermore & Pomraniy (1981)

Rad.-transport: Moments methods

$$
\begin{cases}
\dfrac{1}{c}\dfrac{\partial J_\nu}{\partial t} + \nabla \cdot \boldsymbol{H}_\nu + \alpha_\nu \rho (J_\nu - B_\nu) = 0 \\[2em]
\dfrac{1}{c}\dfrac{\partial H^i_\nu}{\partial t} + \sum_j \dfrac{K^{ij}_\nu}{\partial x_j} + \epsilon_\nu \rho H^i_\nu = 0
\end{cases}
$$

- The moment equations describes the radiation fields, which are related to the energy, energy flux, and radiation pressure
- Consider up to the 1st moment (or M0) —> FLD
- Consider up to the 2nd moment with assumptions of closure (M1, variable Eddington tensor)
- Multi-energy M1 scheme could be very expensive !!!

Rad.-transport: Moments methods



- Three neutrino species
- M1 scheme with 12 energy bins
- 3 (species) x 4 (1 energy + 3D flux) x (12 energy bins) = 144 radiation variables
- Takes > 10M core-hours

Rad.-transport: Ray-tracing methods

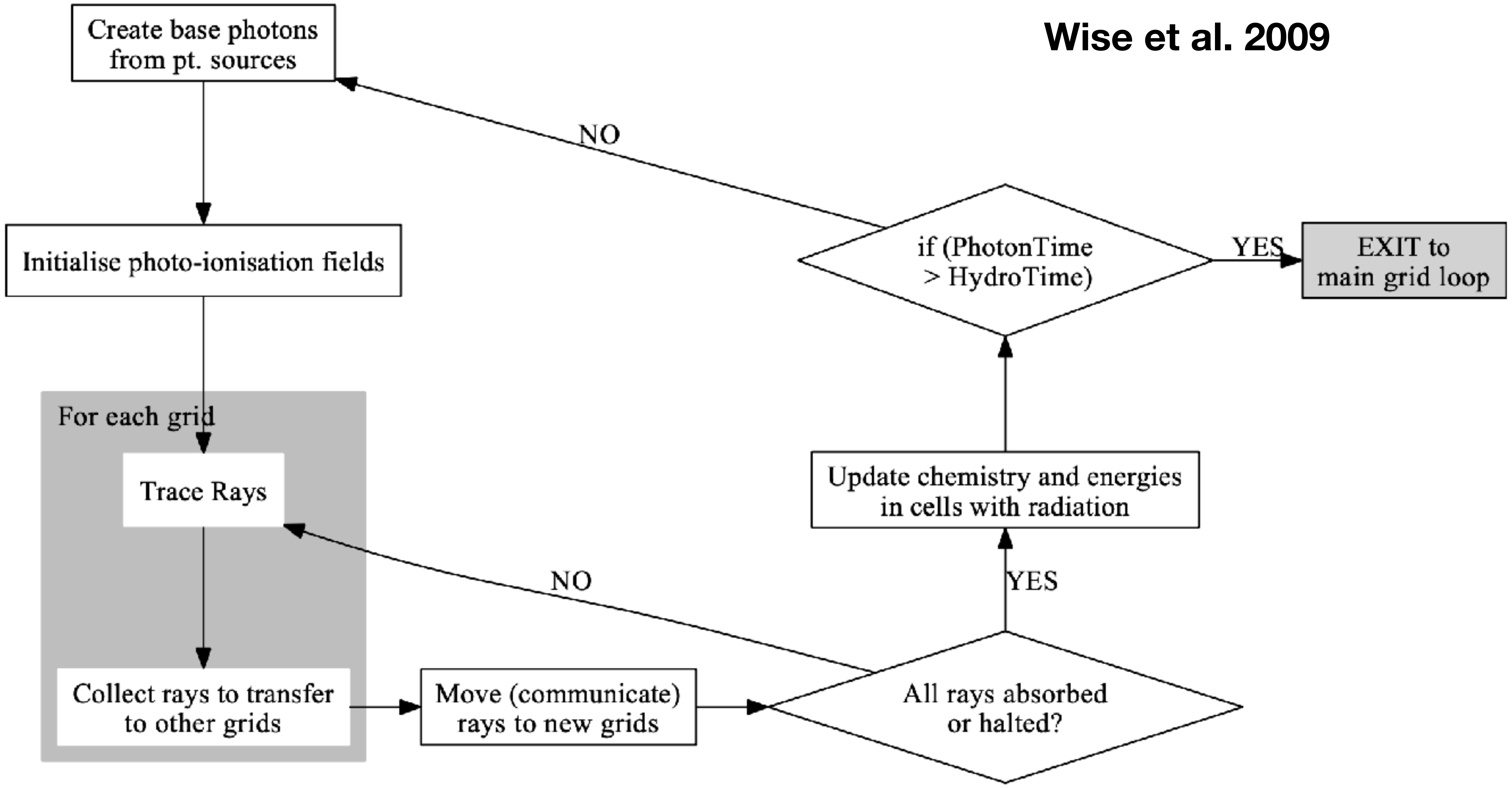
- Usually via post-processing
- Write the transfer equation in Lagrangian coordinates

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

- Shoot a discrete set of light rays from a point source, solving the transfer equation along the path of the light ray.
- The difficulty is then to choose the appropriate number of rays (adaptive ray-tracing method)
- Use Monte-Carlo approach (short or long characteristic)

Rad.-transport: Ray-tracing methods

Wise et al. 2009



Microphysics

- How about opacity, emissivity, and mean free path?
- These depend on complex microphysics which itself depends on the transport of the radiation field
- Usually stored in a table (atomic data, chemical network, eos) ~GB
- Could assume LTE (Saha eq.) or non-LTE (solve reaction network)

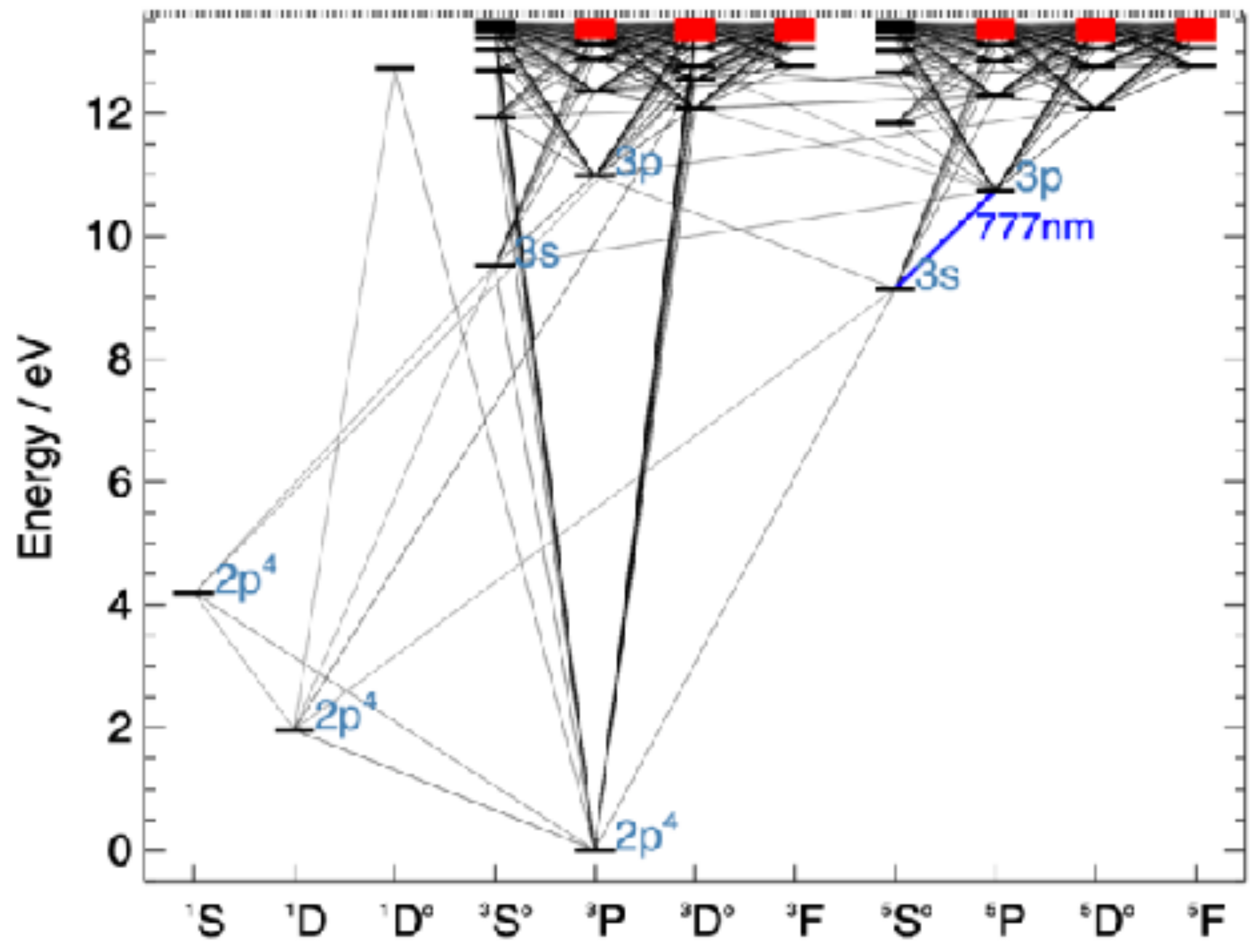


Illustration of a comprehensive "model atom" for neutral oxygen, that describes the structure and radiative and collisional transitions, and is used when calculating the departures from Saha-Boltzmann equilibrium in stellar atmospheres (Amarsi et al., 2018, A&A, 616, 89).

Benchmarks

- Free streaming shadow test

THE ASTROPHYSICAL JOURNAL, 854:63 (19pp), 2018 February 10

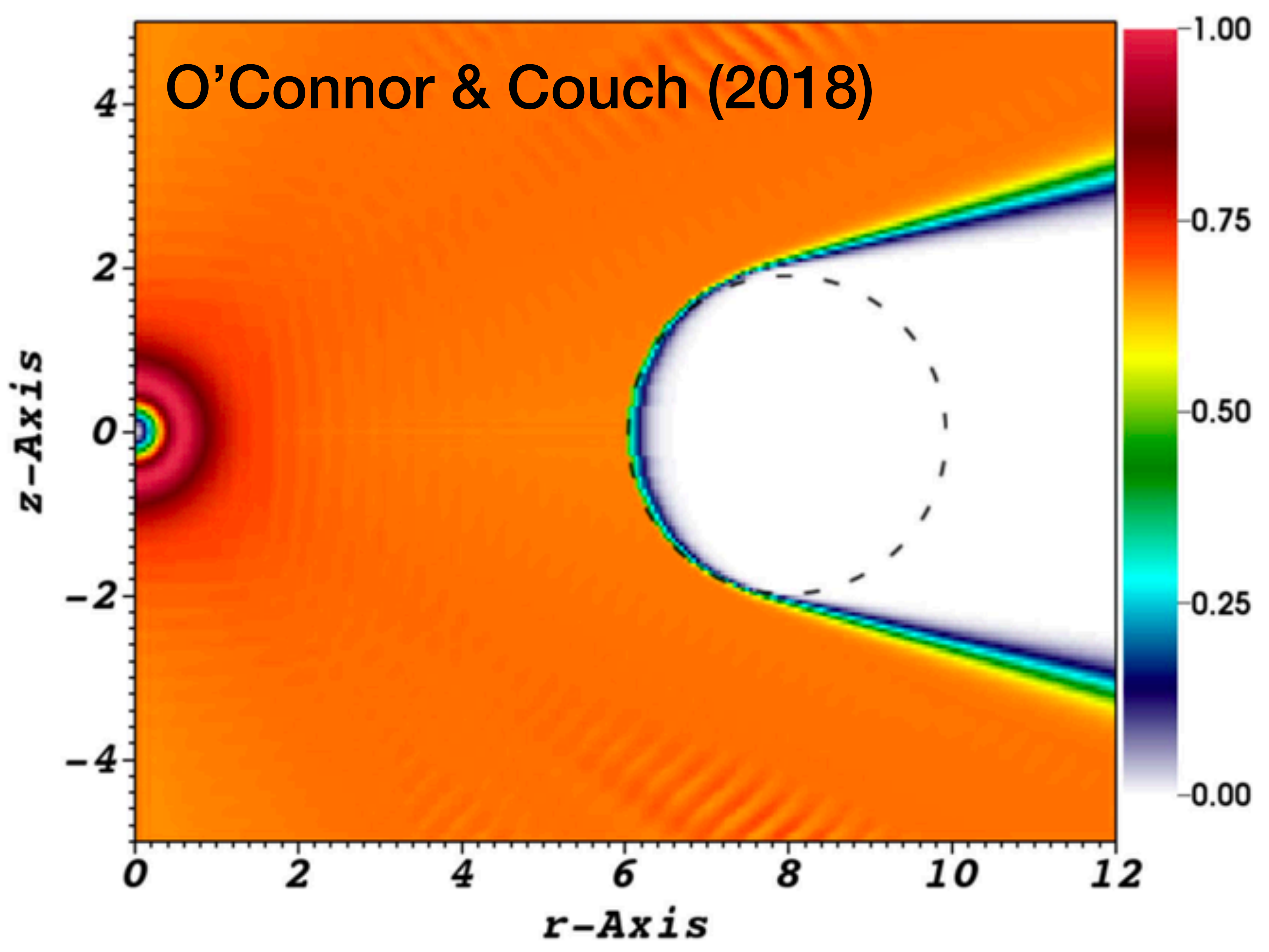
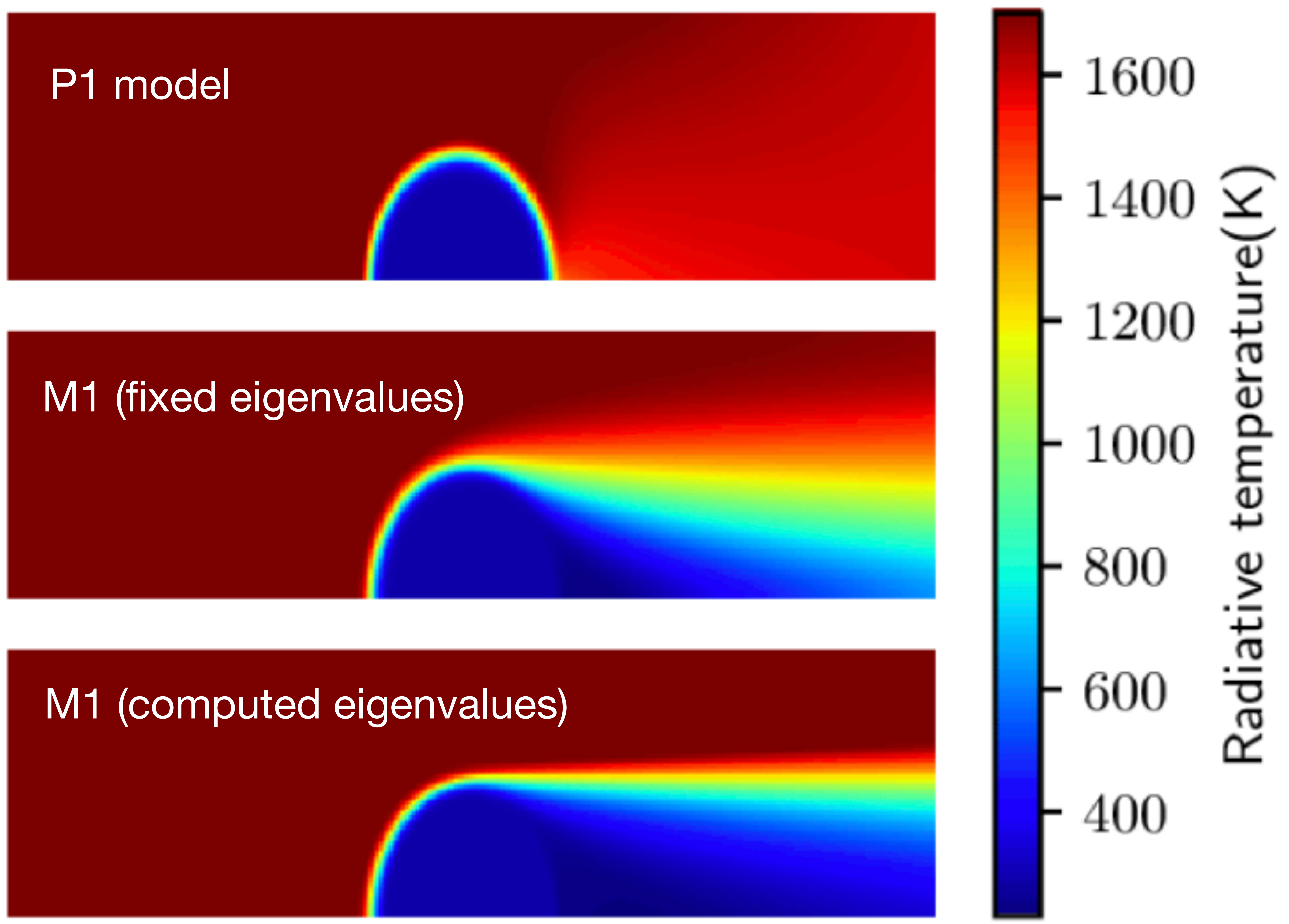


Figure 13. Neutrino energy density multiplied by r^2 in our M1 shadow test in 2D cylindrical coordinates. There is a spherical emission source located at the origin and a perfectly absorbing region (marked by the dashed circle) at $r = 8$ with a radius of 2. This test closely follows the setup of Just et al. (2015).

Bloch et al. (2020)

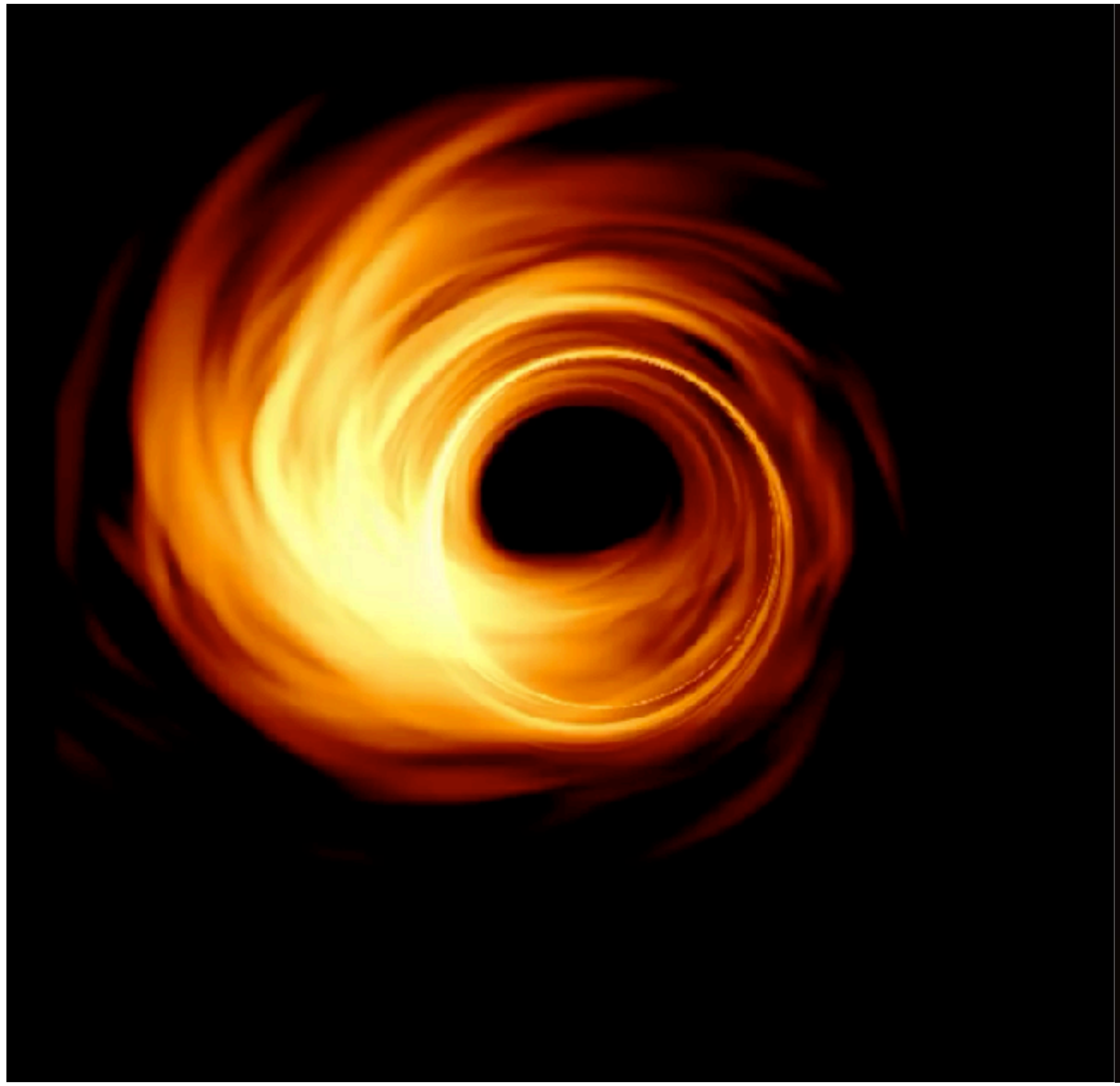


Shadow simulation, showing snapshots of the radiative temperature at time t f = 10 −10 s with different closure relations: P 1 model (upper panel), M 1 model with fixed eigenvalues (middle panel), and M 1 model with computed eigenvalues (lower panel).

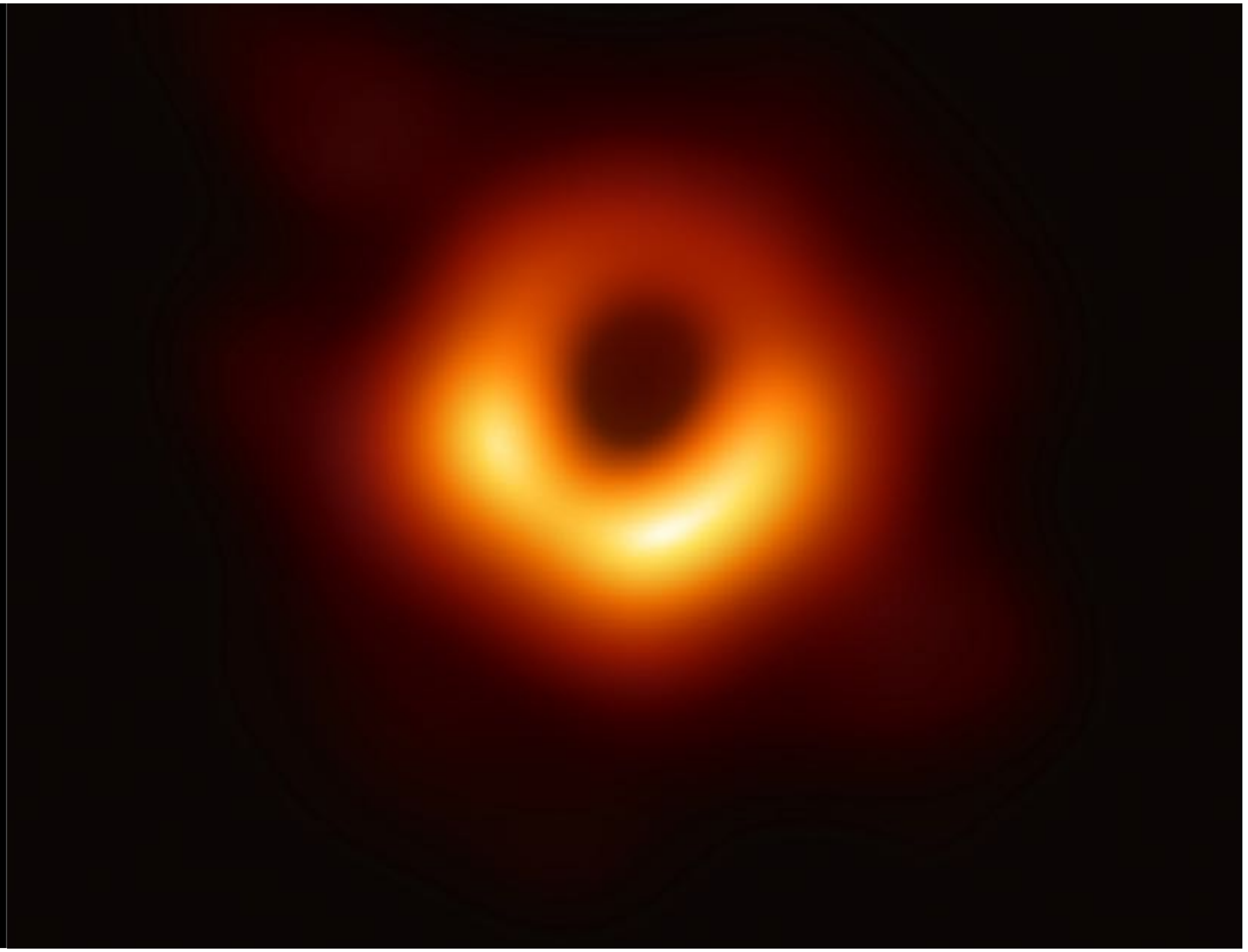
GRMHD + post-precessing ray tracing

Supermassive BH in M87

Recall H.-Y. K. Yang's talk



Shiokawa et al. (2017)



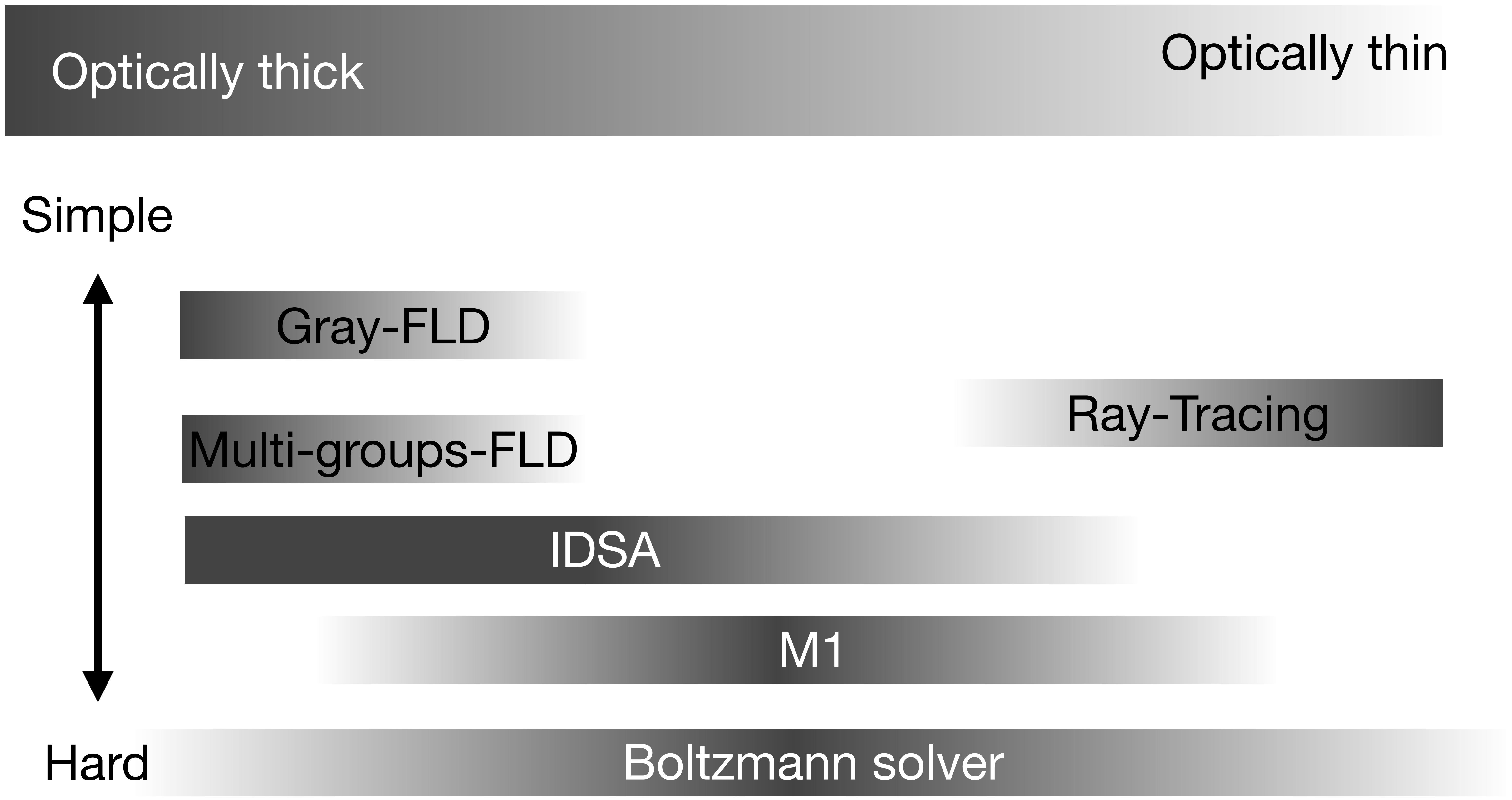
Alberdi et al. (2019)

Numerical approaches - Outline

Non- Transport

- Polytropic EoS
- Tabulated heating /cooling
- Photon/Neutrino leakage

Radiation Transport



	Diffuse Regime	Semi-transparent	Transparent Regime
Boltzmann solver	Truncation errors in flux		Inefficient ang. res.
Flux-limited diffusion		Flux factor estimated	Flux factor unknown
Ray-tracing	Short mean free path	Limited by reaction rates	

The ideal algorithm combines the three green regions.
However, it might be too complicated.
Alternatives: variable Eddington factor method; M1, and the IDSA

[Adjusted from M. Liebendörfer]

Summary

ray-tracing
Spontaneous Emission
Black hole shadow
flux-limited diffusion
Star formation
atomic data
LTE Scattering process
Monte Carlo
Boltzmann equation
Radiation transport
IDSA
Closure
microphysics
Hyperfine Splitting
Stimulated emission
neutrino radiation
Variable Eddington Tensor
Supernovae
Zeeman splitting
Absorption coefficient
Multi-group flux limited diffusion
moment methods
neutrino interactions

Application: Supernova with Neutrino Transport

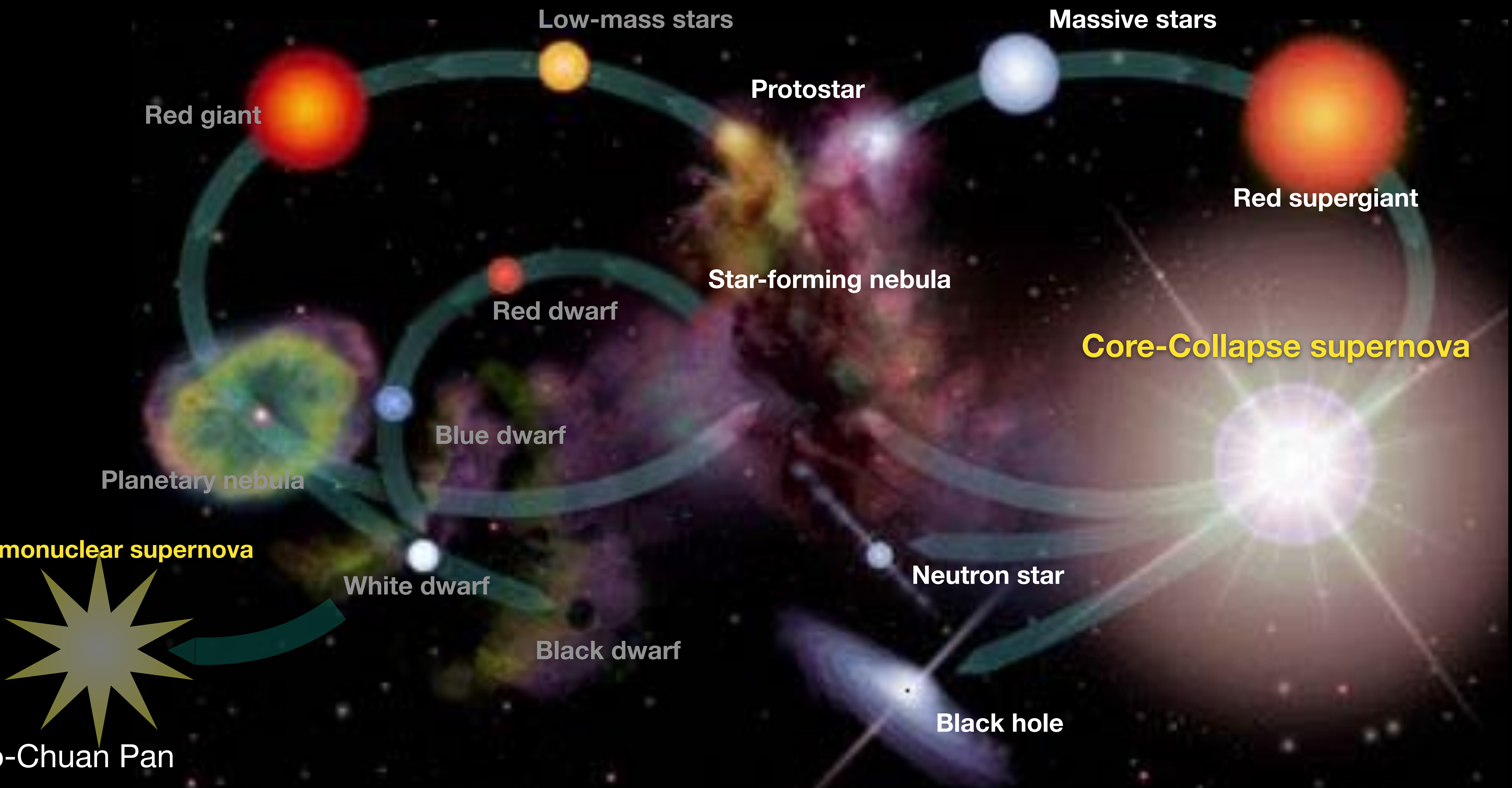
Application: Core-Collapse supernovae

- Neutrino Transport (not photons)

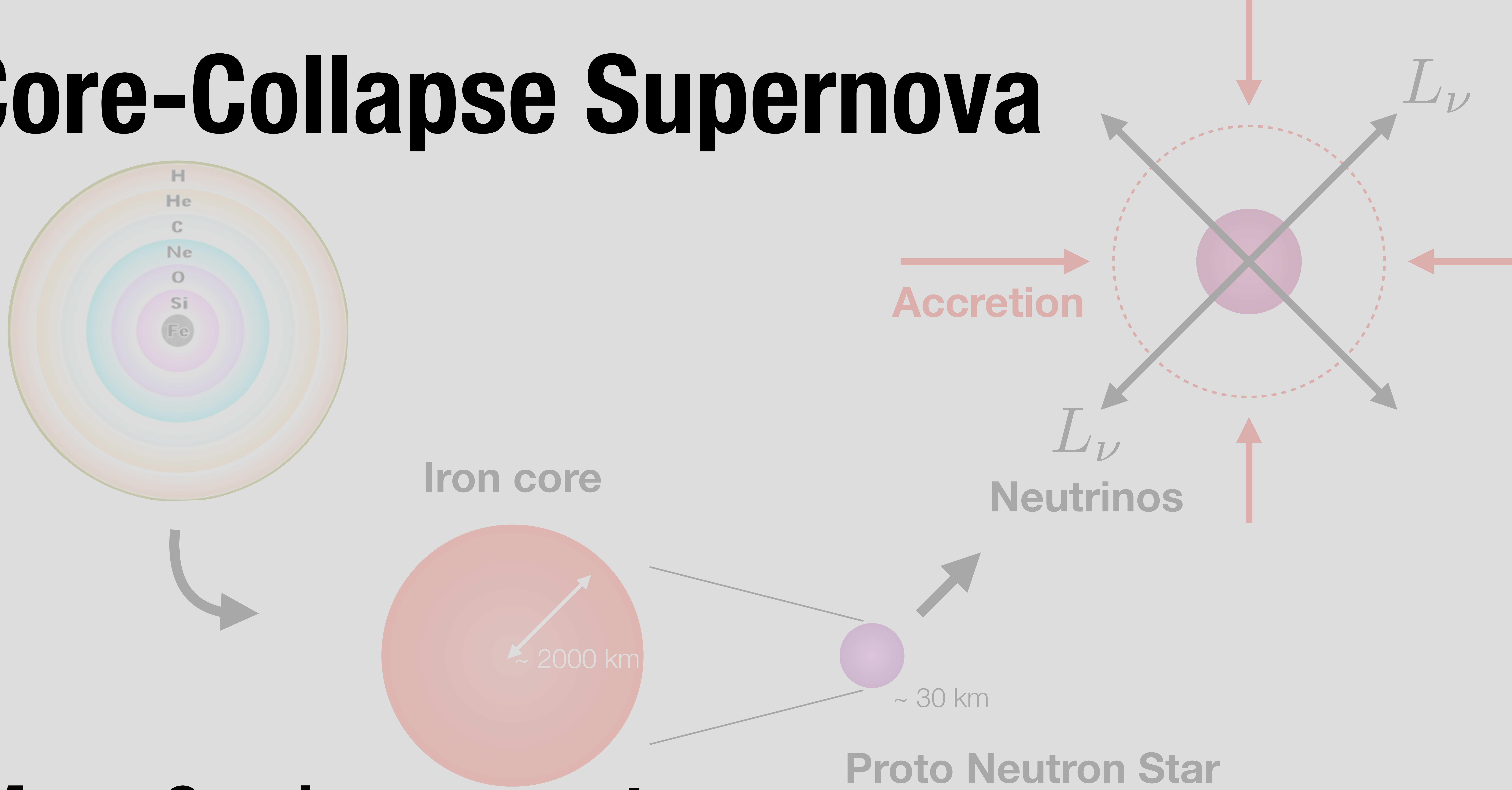
Recall H-Y Karen Yang's talk

- Not only supernovae, but also neutron star mergers, … etc.
- Neutrinos are fermions (photons are bosons)
- Neutrinos have difference flavors (and anti-neutrinos)
- Relativistic effects can not be ignored
- Complicated neutrino interactions (oscillations?)
- Cover both optically thick and thin

Stellar evolution 100

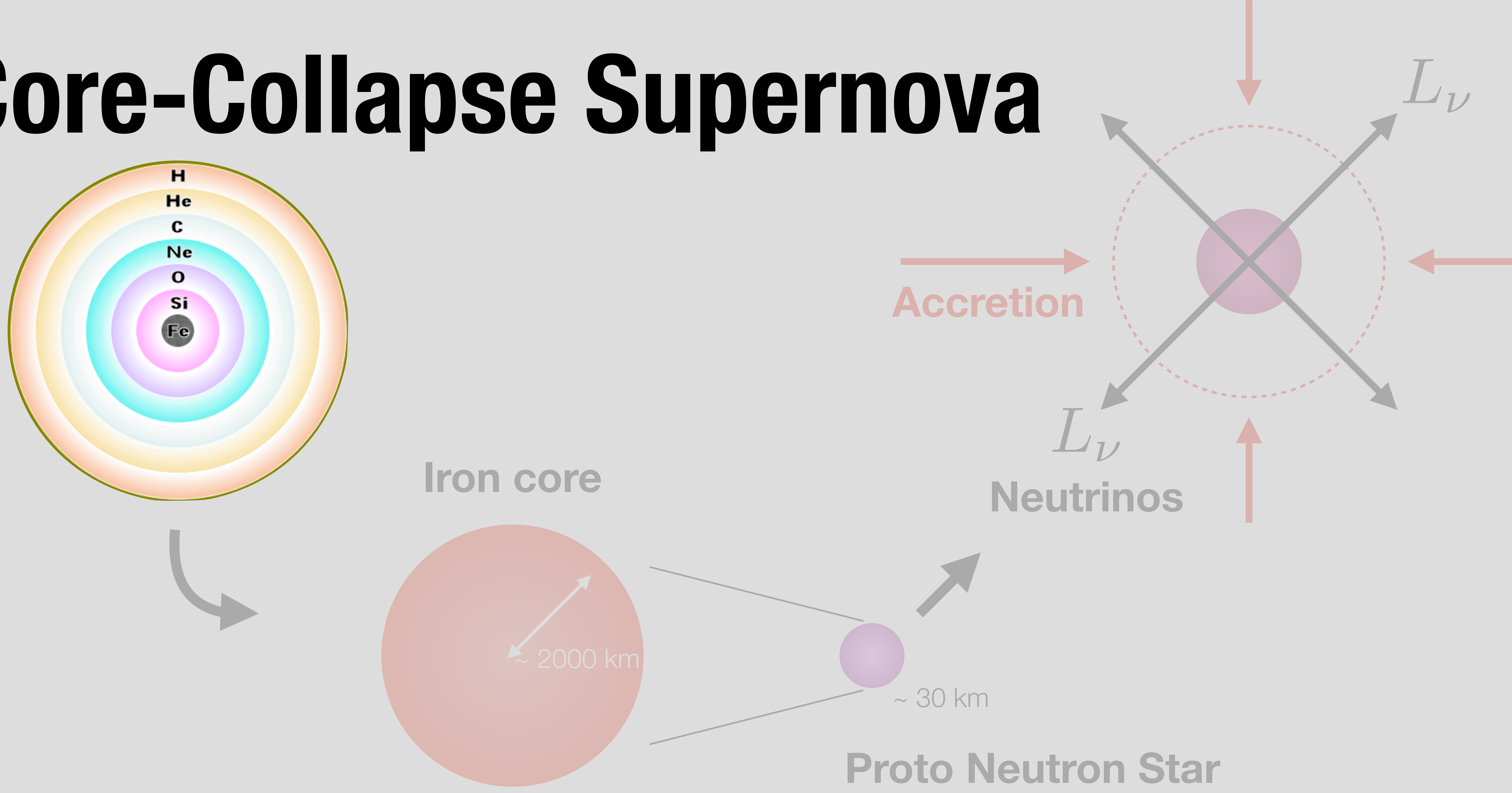


Core-Collapse Supernova



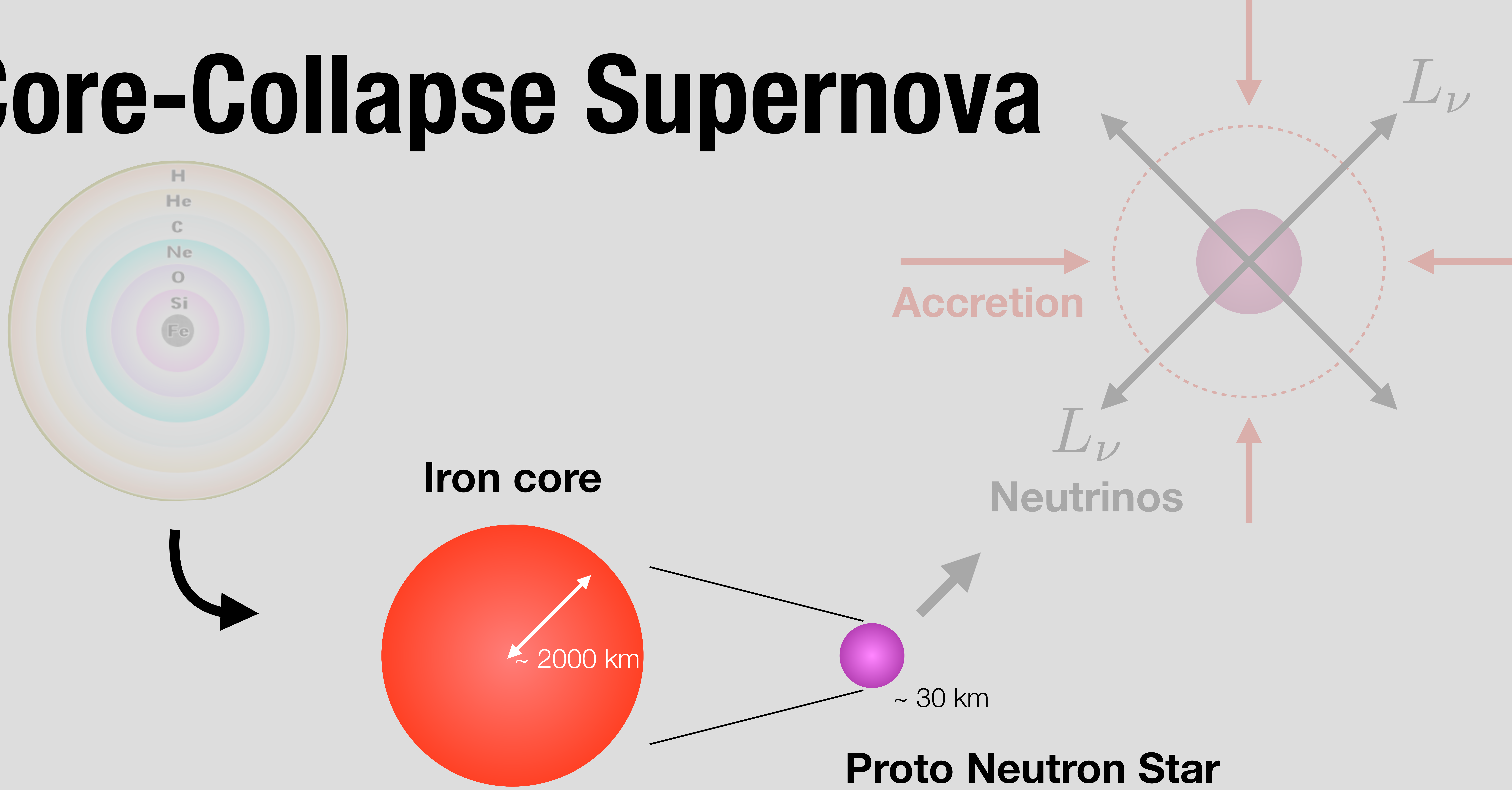
M > ~ 8 solar mass stars

Core-Collapse Supernova



Kuo-Chuan Pan

Core-Collapse Supernova



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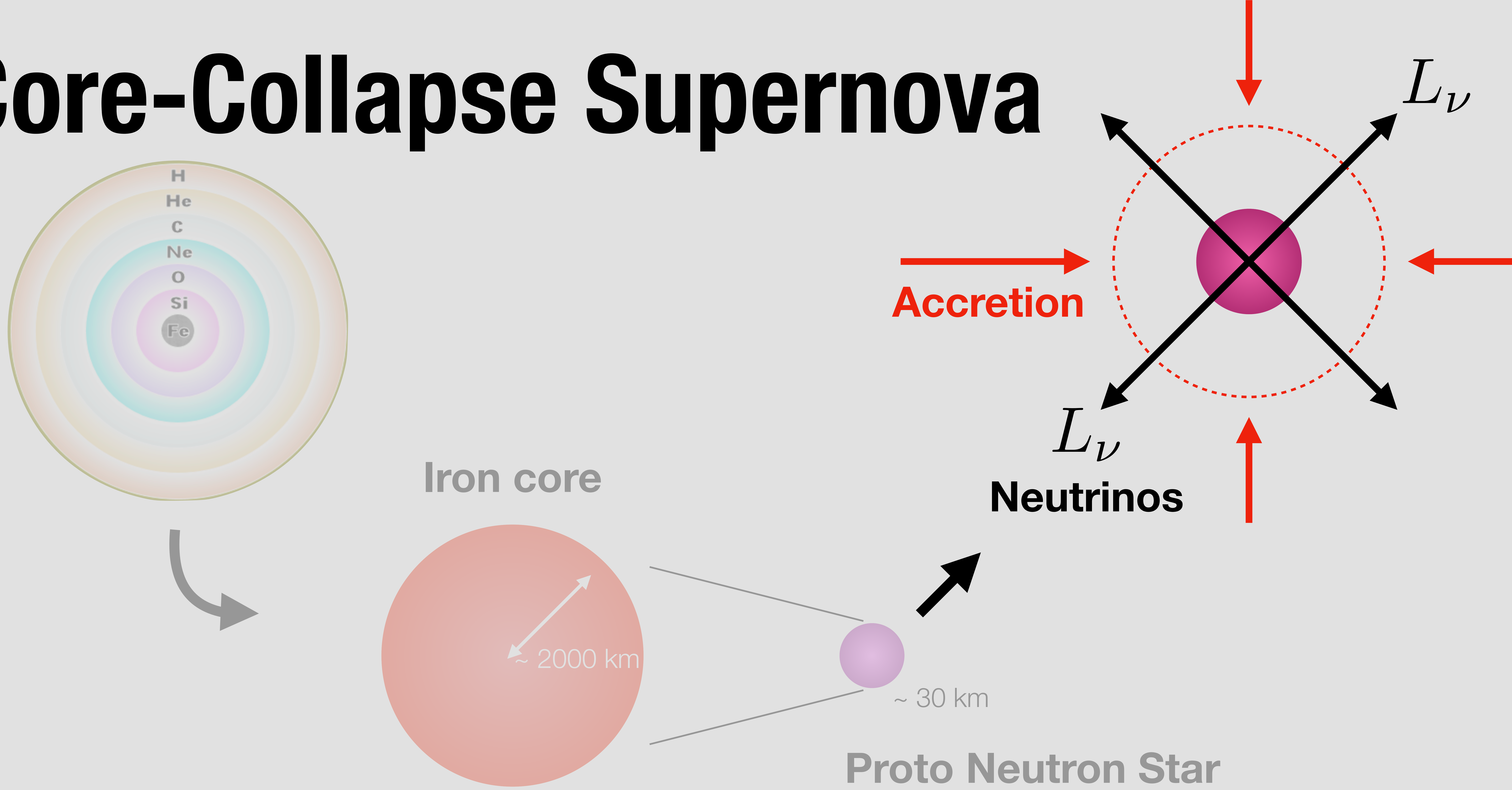
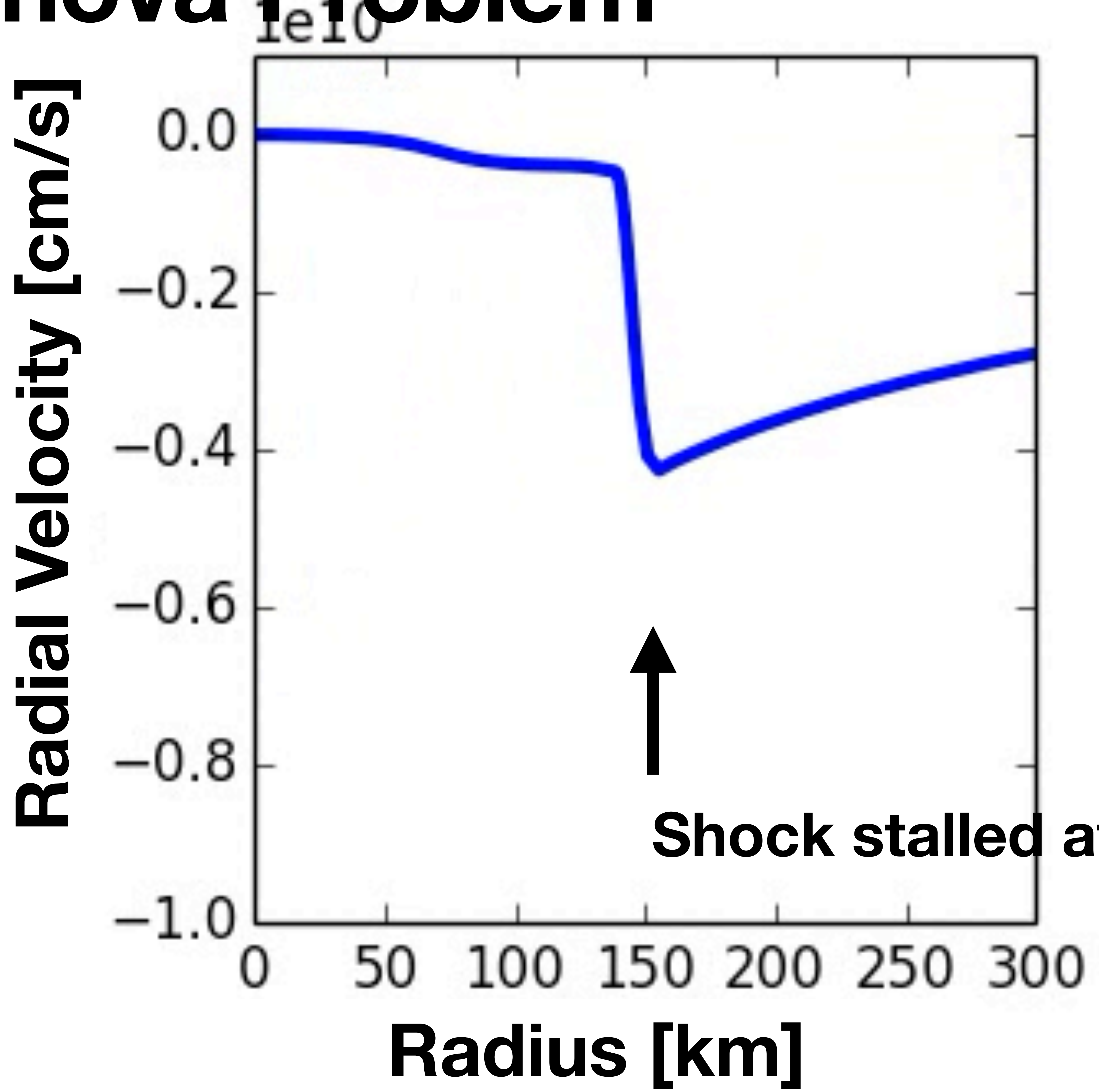
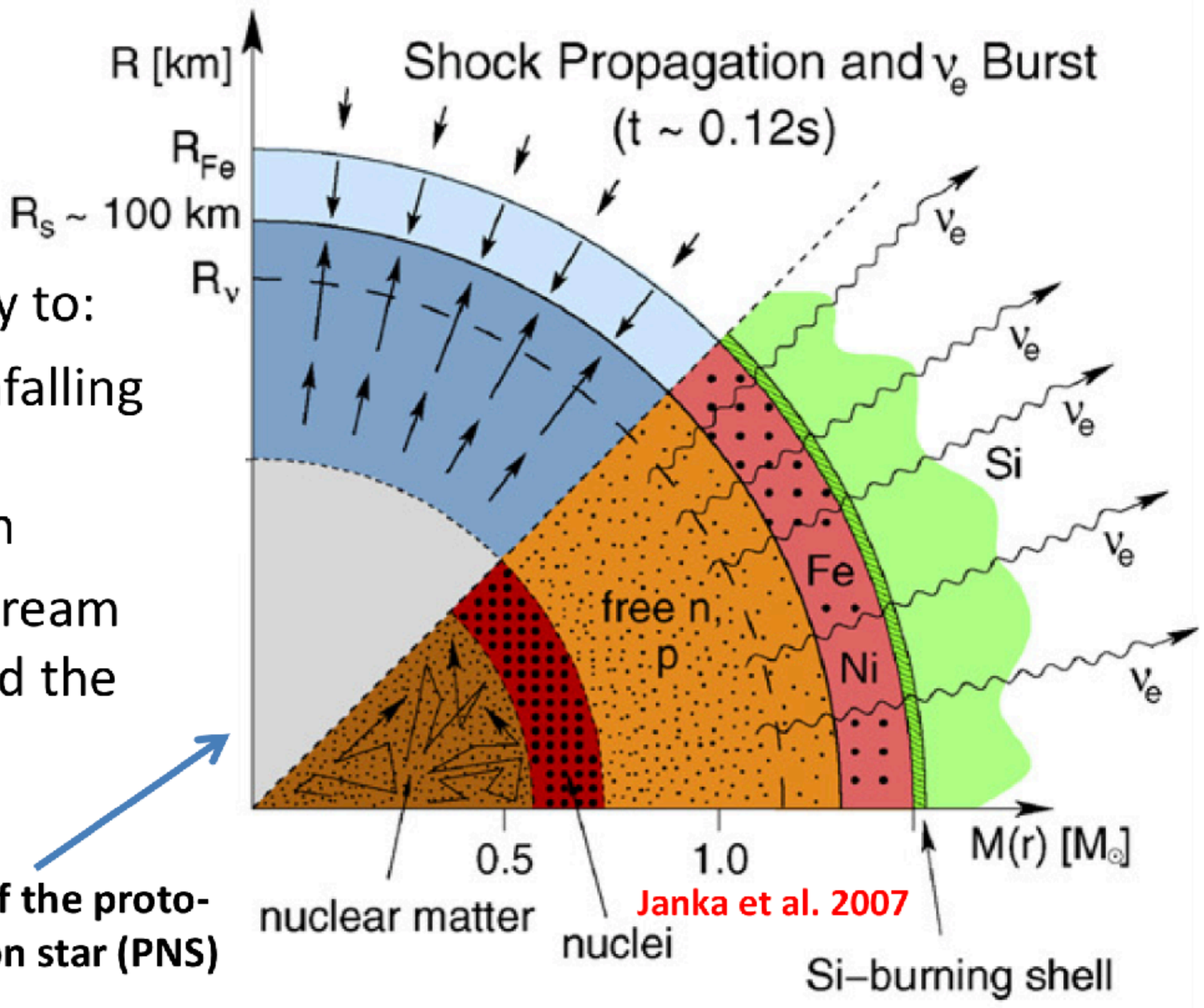


The Supernova Problem



Why does the shock stall?

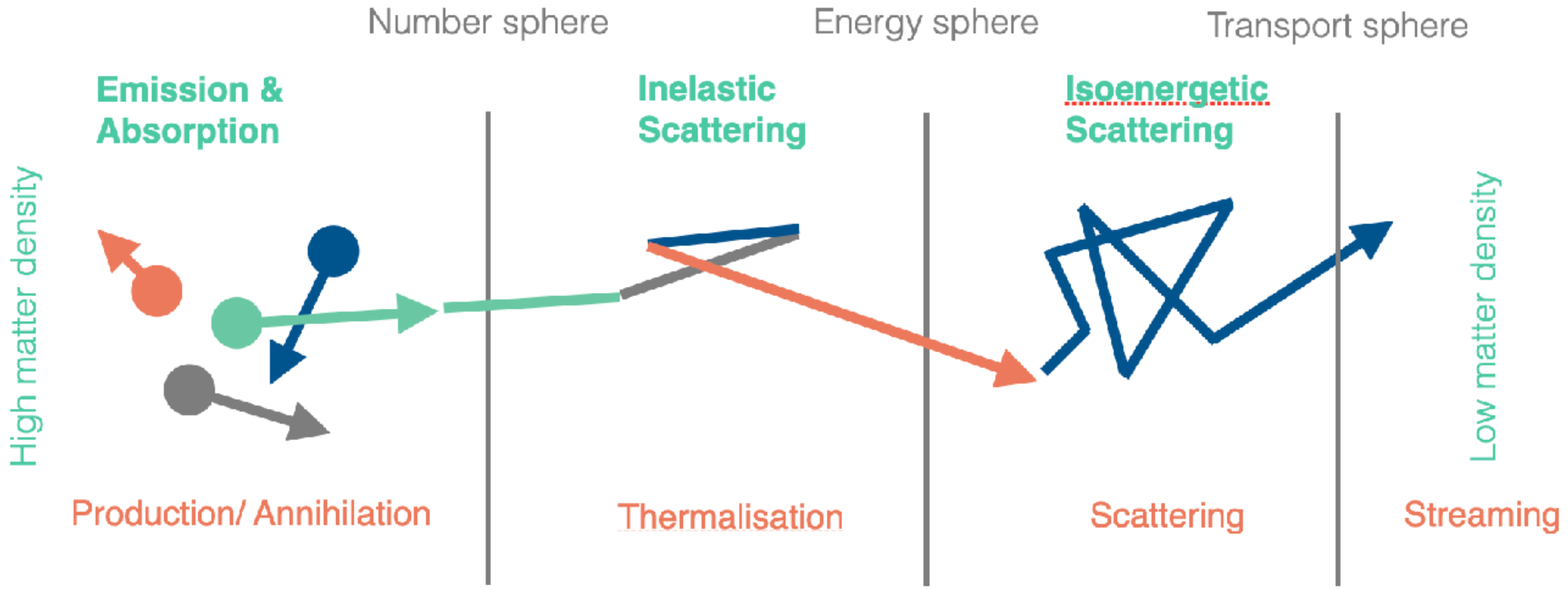
- Shock loses energy to:
  - Dissociation of infalling heavy nuclei: ~8.8 MeV/baryon
  - Neutrinos that stream away from behind the shock.



Janka et al. 2007

Inner core -> Core of the proto-neutron star (PNS)

Neutrino Matter Interactions



Electron/ positron capture

$$\nu_e + n \rightleftharpoons e^- + p$$
$$\bar{\nu}_e + p \rightleftharpoons e^+ + n$$
$$\nu_e + (A, Z) \rightleftharpoons e^- + (A, Z+1)$$

NN bremsstrahlung (Thompson+02)

ve pair —> vu pair (Buras+03)

Neutrino-electron scattering

$$\nu + e \rightleftharpoons \nu + e$$

Inelastic neutrino-nucleus scattering

Elastic coherent scattering of neutrinos on nuclei

$$\nu + (A, Z) \rightleftharpoons \nu + (A, Z)$$

Neutrino-nucleon scattering

$$\nu + N \rightleftharpoons \nu + N$$

[Adjusted from M. Liebendörfer]

Neutrino Luminoisty

- Optical depth

$$\tau_\nu(r) = \int_\infty^r \frac{1}{\lambda_\nu} dr'$$

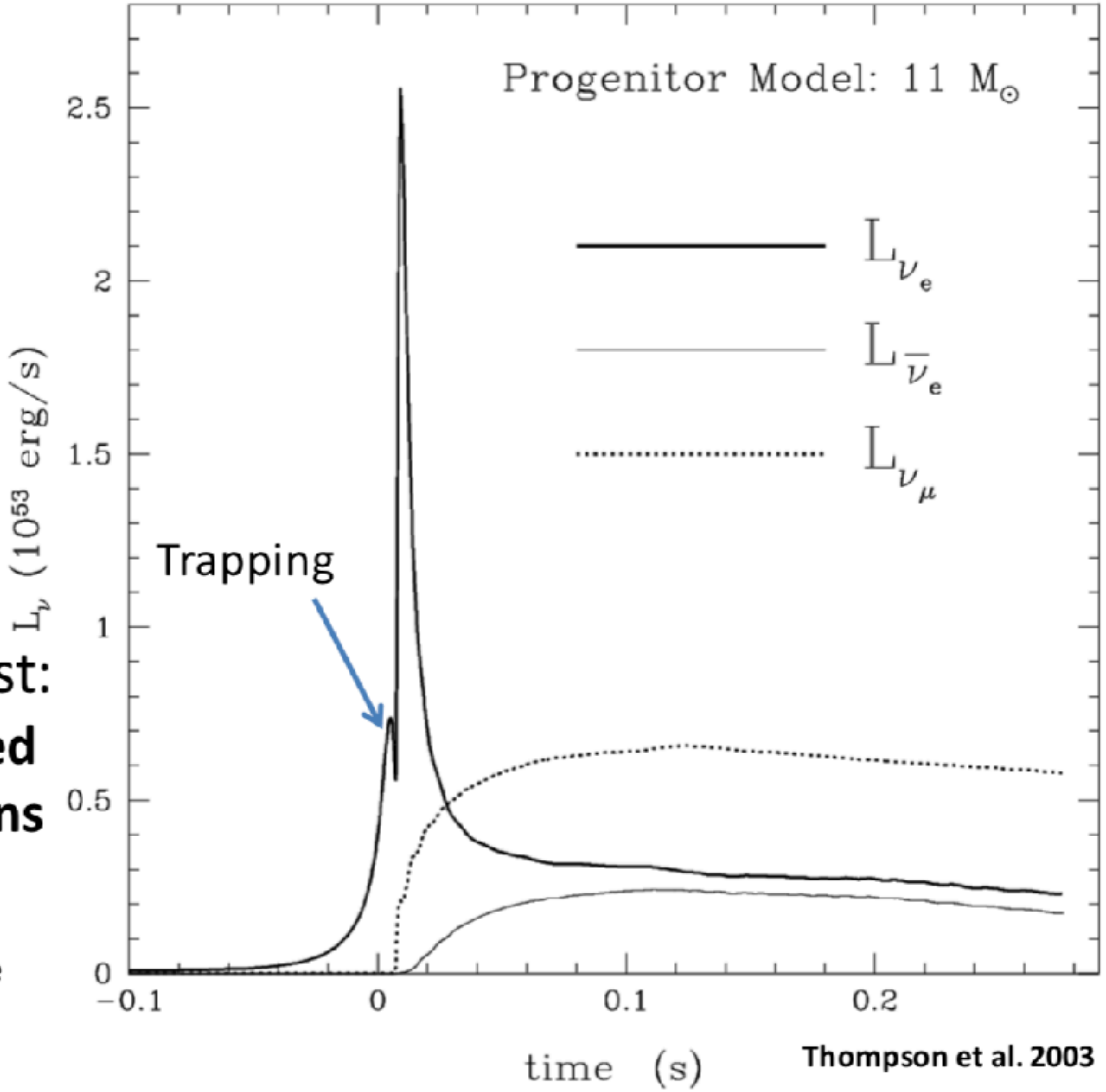
- Neutrinosphere:

$$R_\nu = R\left(\tau_\nu = \frac{2}{3}\right)$$

Depends on $(\epsilon_\nu)^2$

- Postbounce neutrino burst:

Release of neutrinos created by e$^-$ capture on free protons in shocked region when shock 'breaks out' of the ν_e neutrinospheres.



Thompson et al. 2003

Supernova mechanism

- Collapse to neutron star ~ 300 B
- 1B kinetic and internal energy of the ejecta (or ~10B for hypernova)
- 99% of the energy is radiated as neutrinos over hundreds of seconds as the protoneutron star cools
- Explosion mechanism must tap the gravitational energy reservoir and convert the necessary fraction into energy of the explosion.

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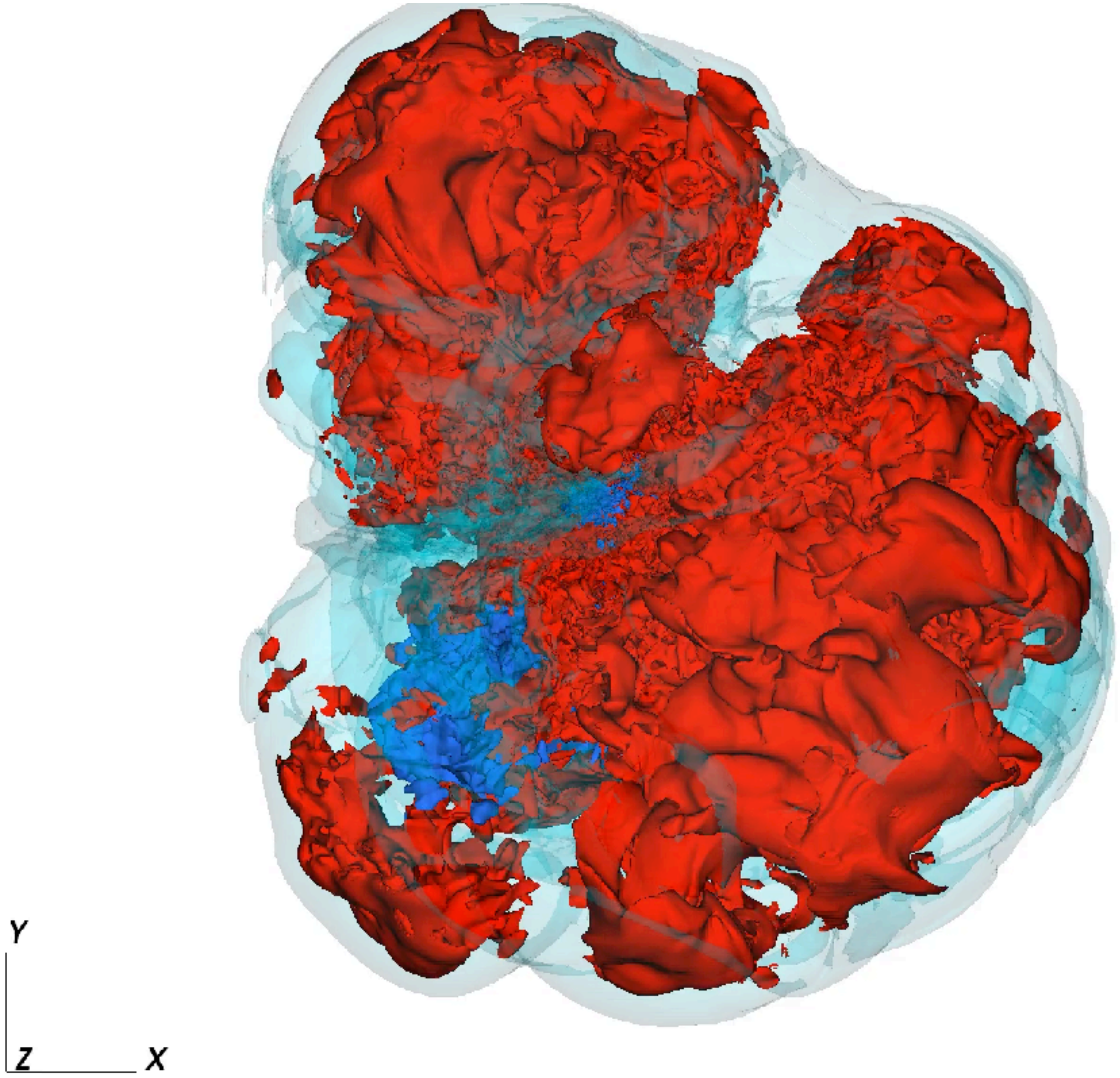
MOST 科技部
Ministry of Science and Technology

NARLabs 財團法人國家實驗研究院
NCHC 國家高速網路與計算中心
National Center for High-performance Computing

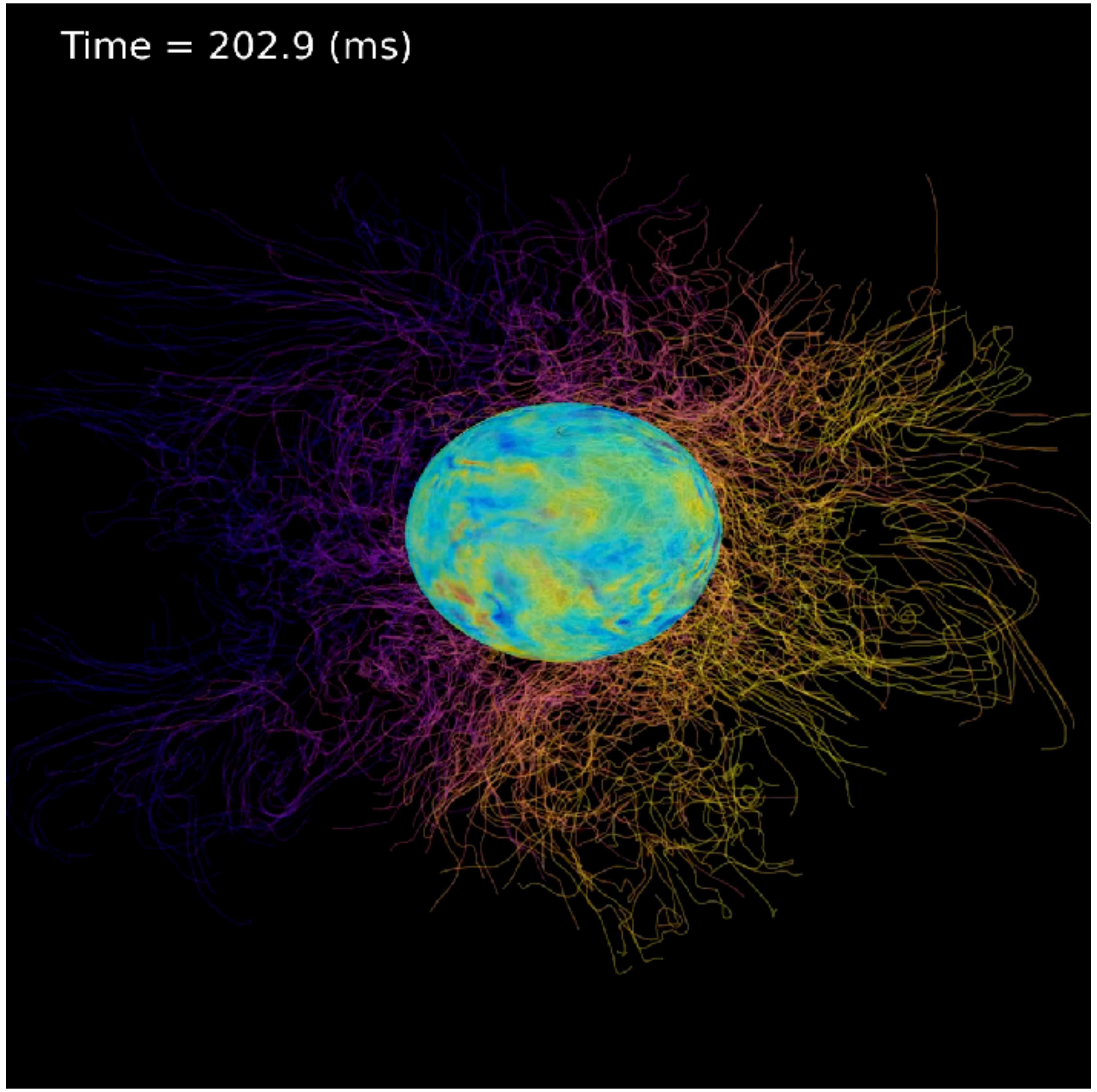
Core-Collapse Supernova Simulation

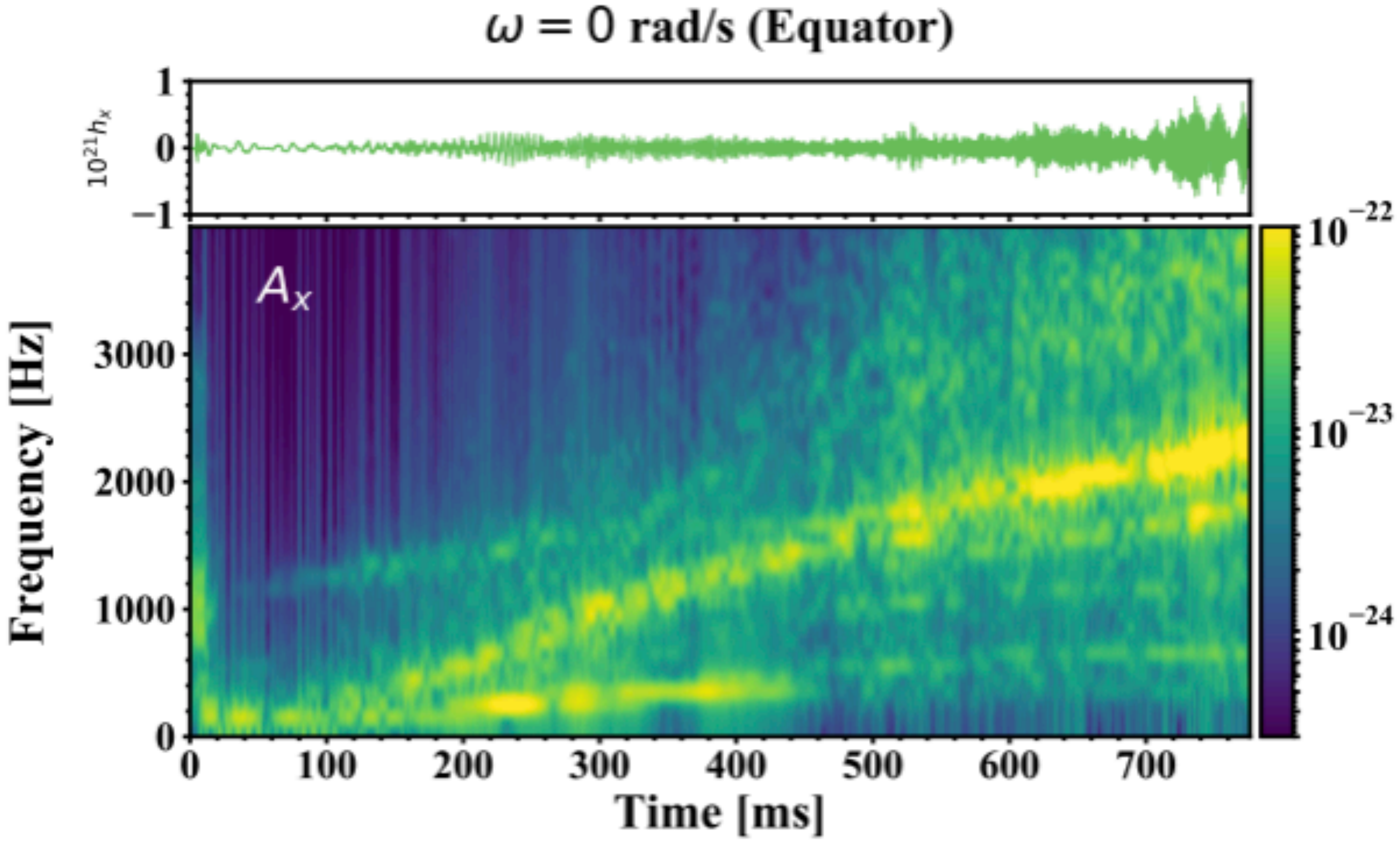
Visualization: Kuo-Chuan Pan (潘國全)
Department of Physics
Institue of Astronomy
National Tsing Hua University, Taiwan

Compositions



Gravitational wave from CCSNe





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