STAR FORMATION

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Department of Physics, National Sun Yat-sen University

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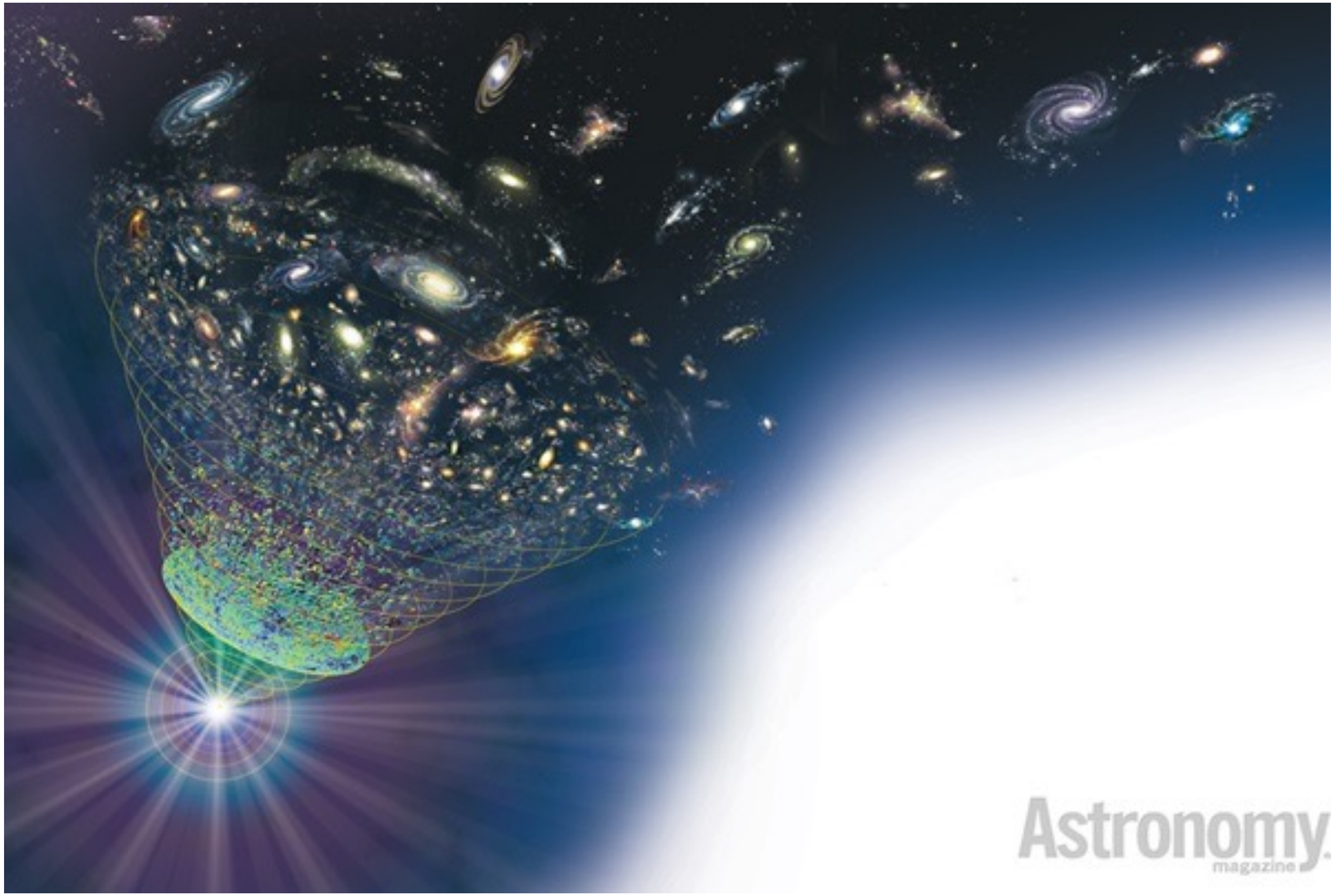
Status of this Field

1. You can be known for discovering or addressing something fundamental
2. There are more questions than theories and answers
3. The duty-cycle of making your own hypothesis and then test it is short
4. This field is under-credited and is presently not particularly hot ☹

Why Star-Formation Matters



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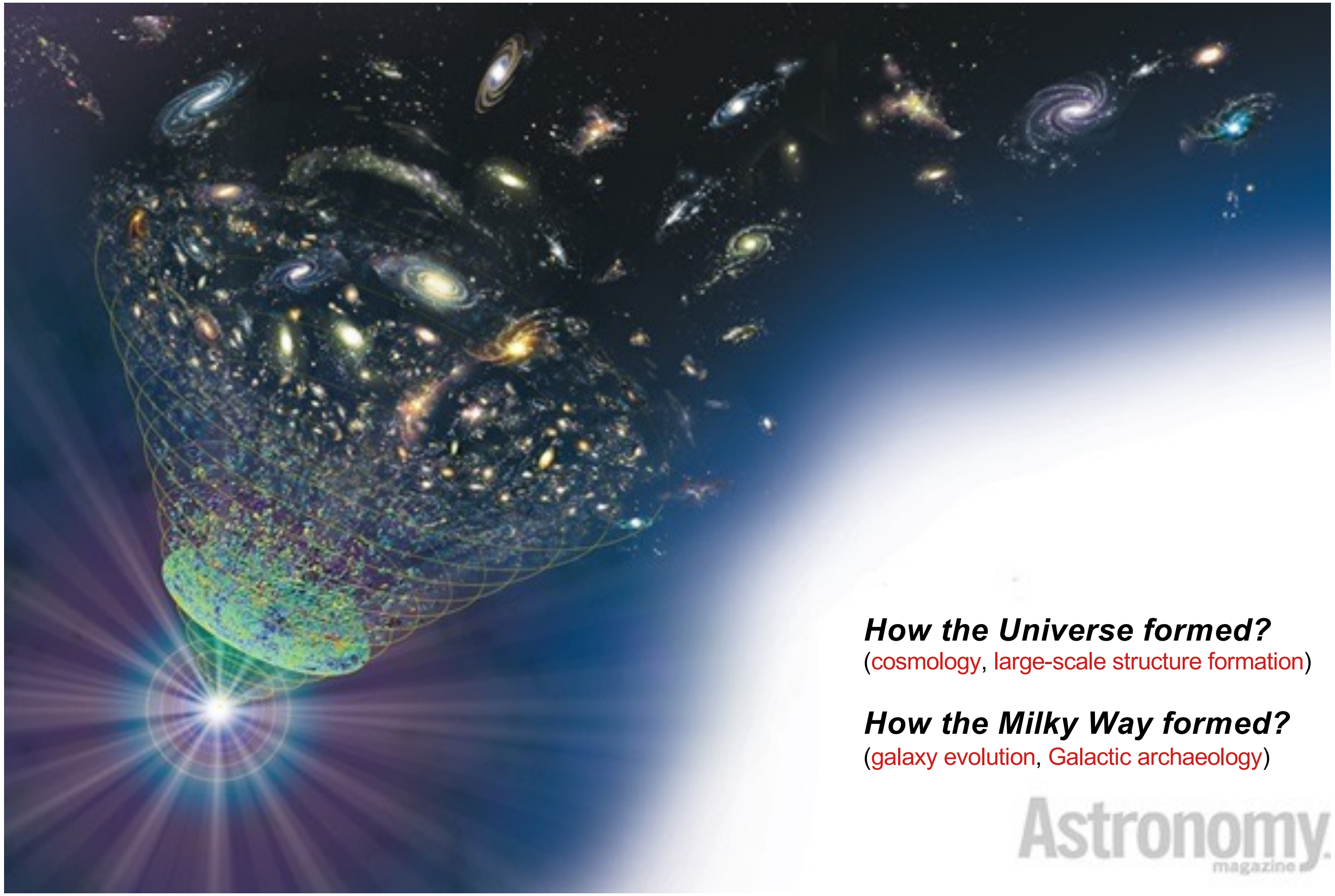
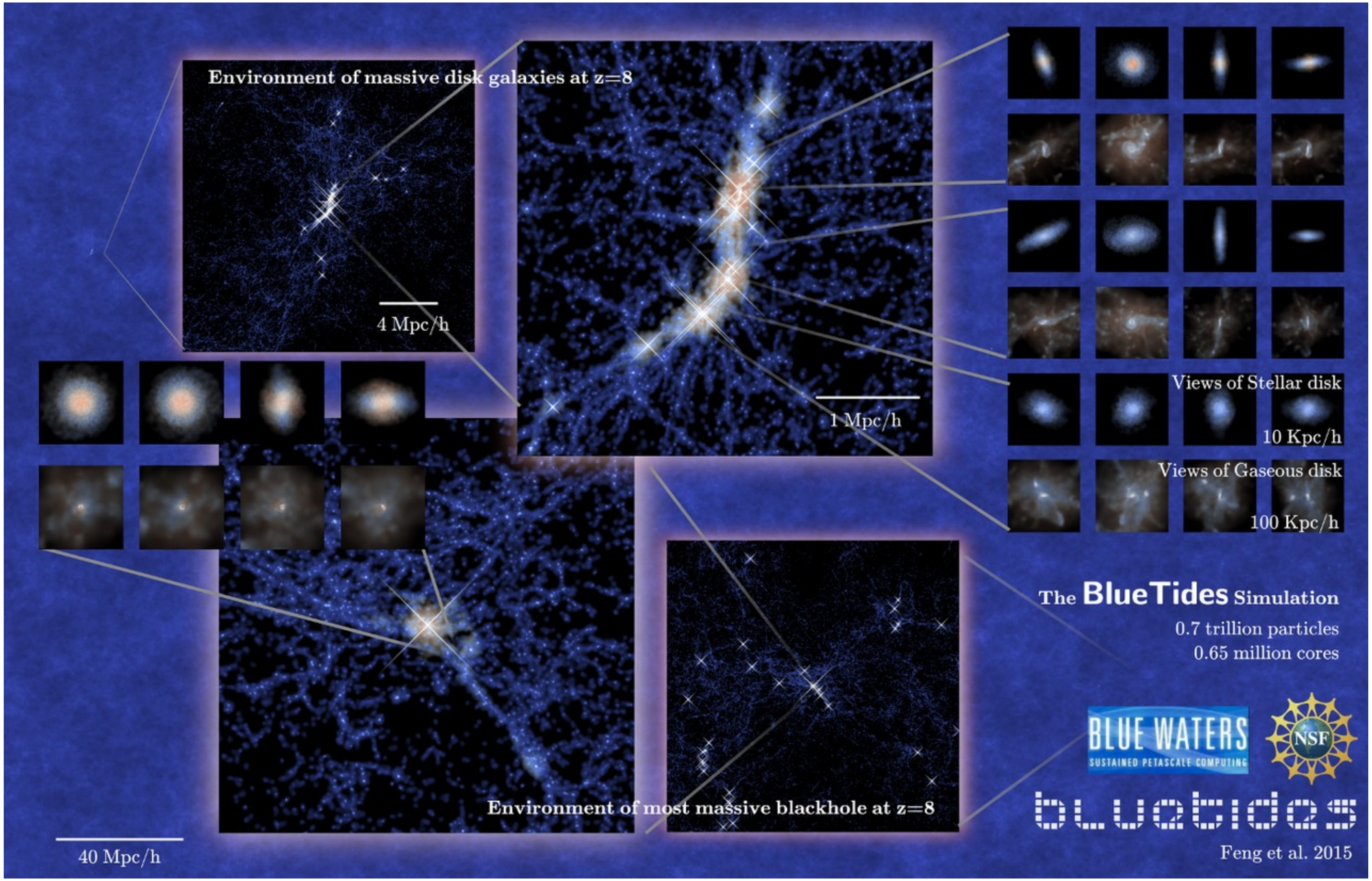


How the Universe formed?
(cosmology, large-scale structure formation)

How the Milky Way formed?
(galaxy evolution, Galactic archaeology)

Stars Make Galaxies and the Parent Dark Halo Visible to us



Uncertain Baron Physics

(1) Halo Occupation Distribution (2) Kennicutt-Schmidt Law (3) Gao-Solomon Relation

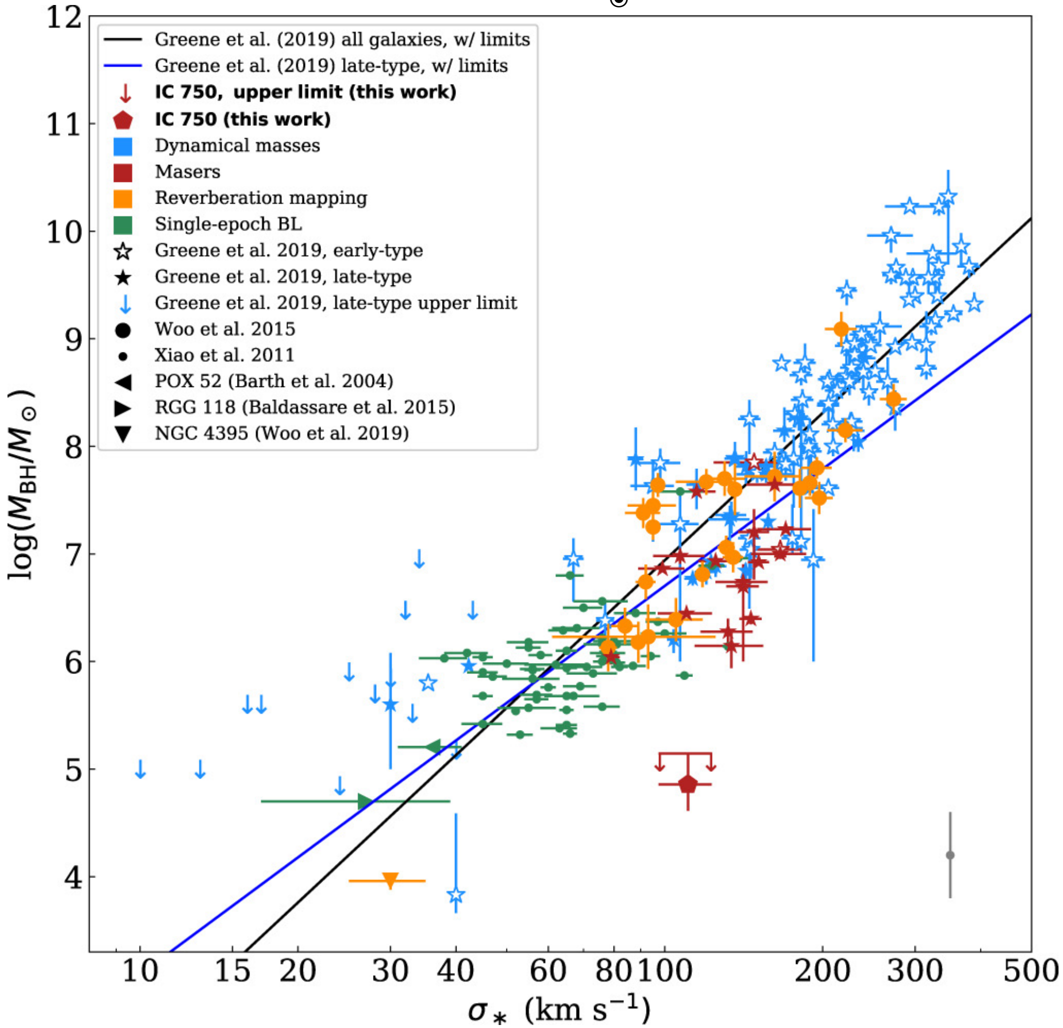
Uncertain Baron Physics

(1) **Halo Occupation Distribution** (2) Kennicutt-Schmidt Law (3) Gao-Solomon Relation

(i) How sub-halos (candidates of galaxies ?) accrete baryonic material? And how feedback and other physics quench the accretion (e.g., AGN feedback, ram pressure stripping in a galaxy cluster). The observations of inter-galactic medium (IGM) are to address these issues.

(ii) How the first-generation stars (also known as the Population III stars) formed, and what were their roles in shaping the visible universe?

M-σ relation: $\frac{M}{10^8 M_\odot} \approx 1.9 \left(\frac{\sigma}{200\ km\ s^{-1}}\right)^{5.1}$



Zaw et al. 2020, ApJ, 897, 11

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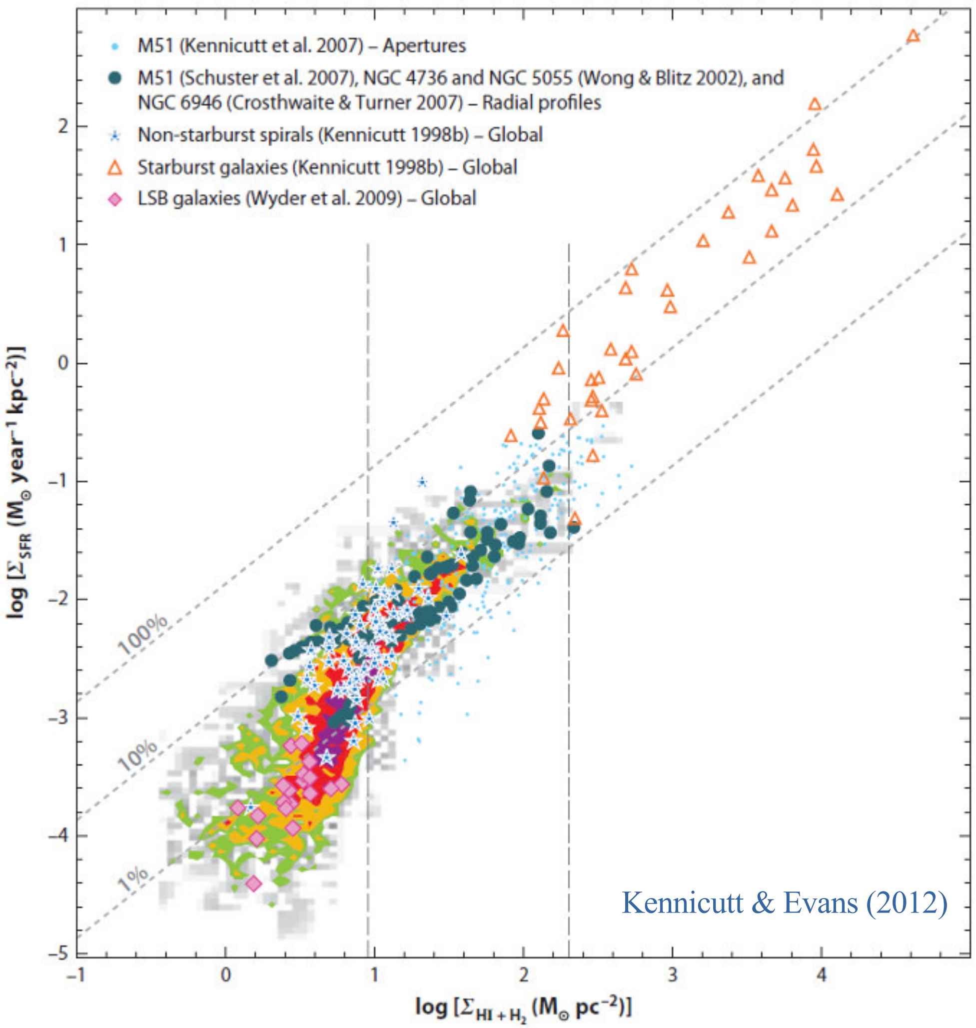

$$\Sigma_{SFR} = 2.5\times10^{-4}\left(\frac{\Sigma_{gas}}{1\ M_\odot\ pc^{-2}}\right)^{1.4} M_\odot yr^{-1} kpc^{-2}$$

Kennicutt et al. 1998, ApJ, 498, 541

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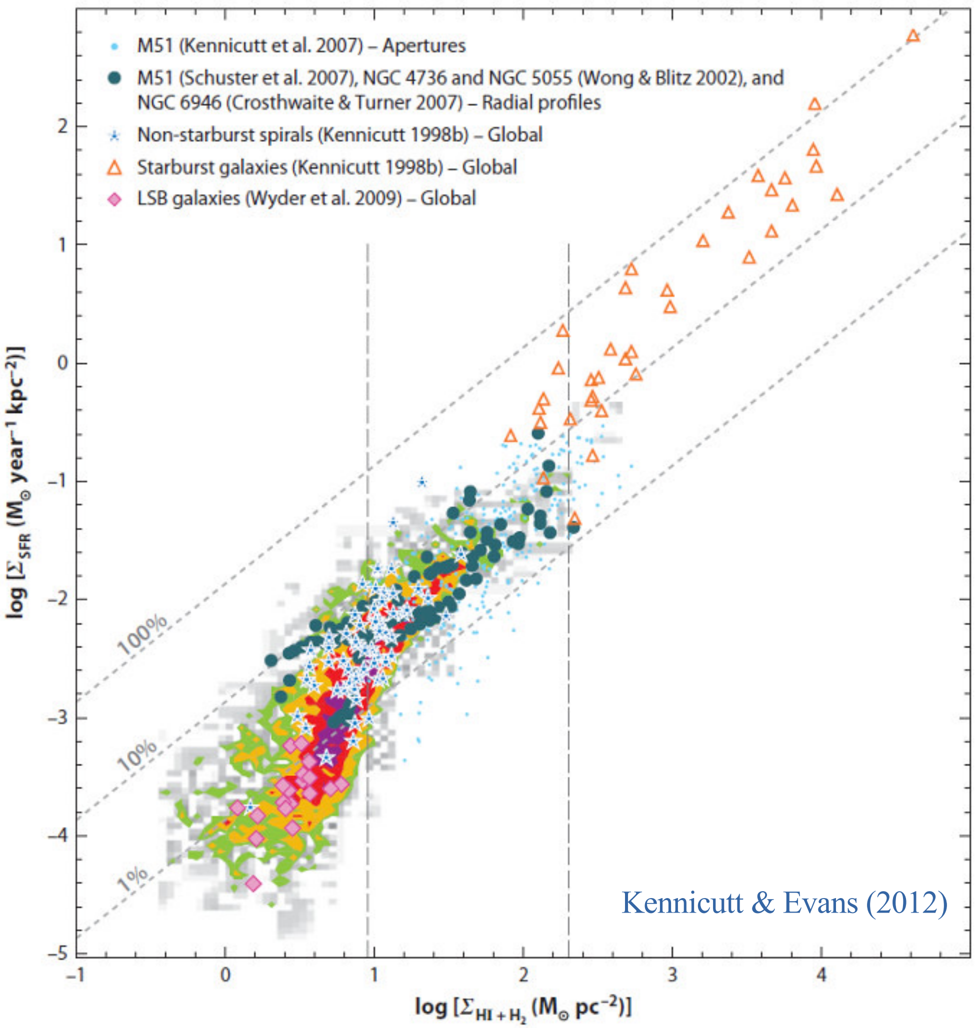

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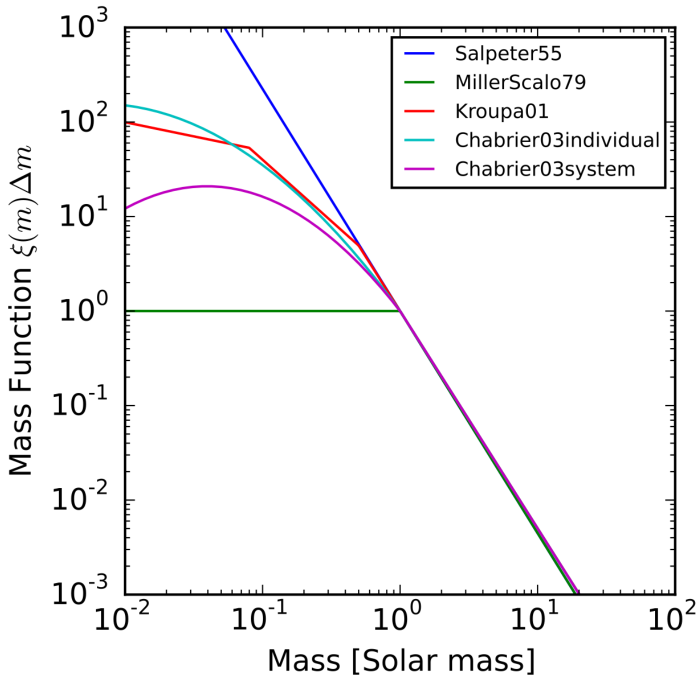
Stellar Initial Mass Function



https://en.wikipedia.org/wiki/Initial_mass_function

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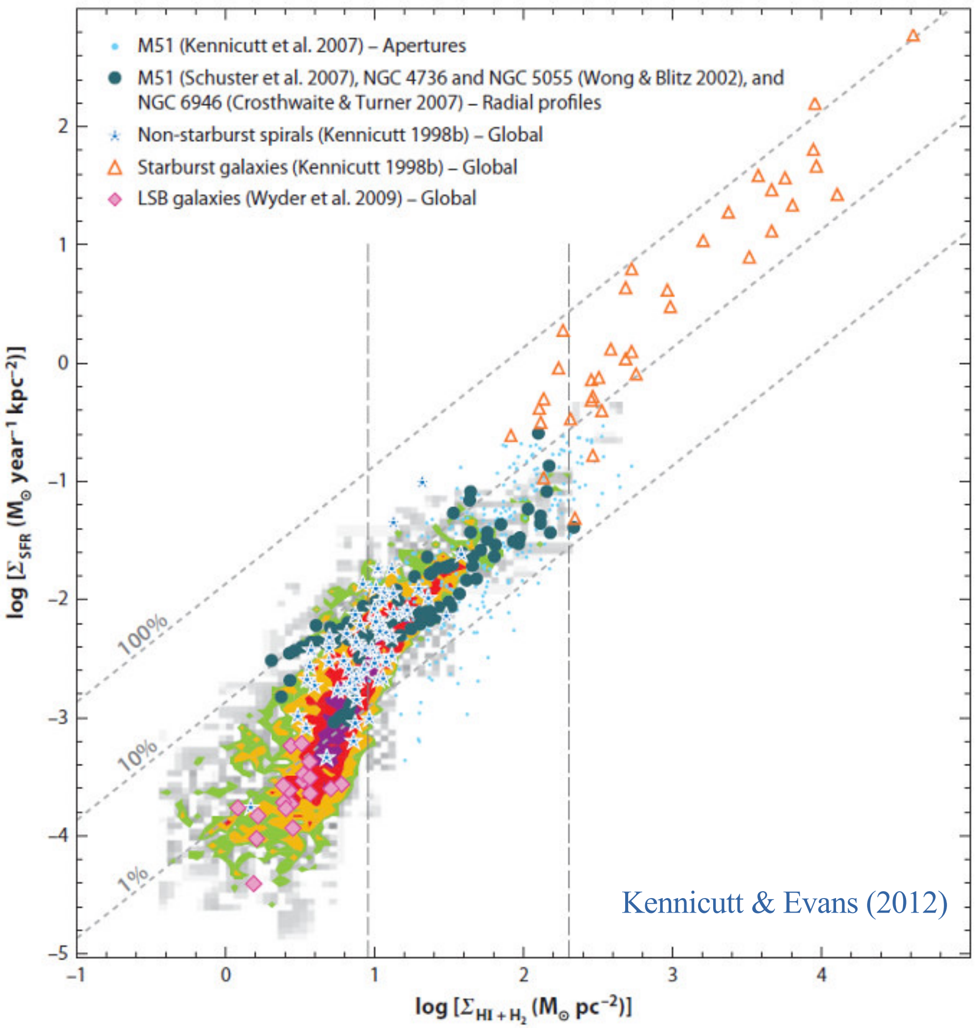

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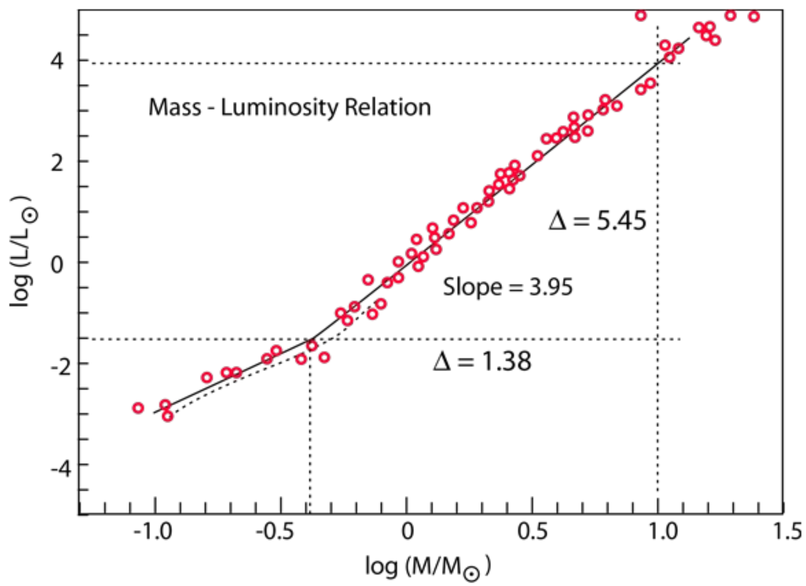
Stellar Mass-Luminosity Relation



Kuiper et al. 1938, ApJ, 88, 472

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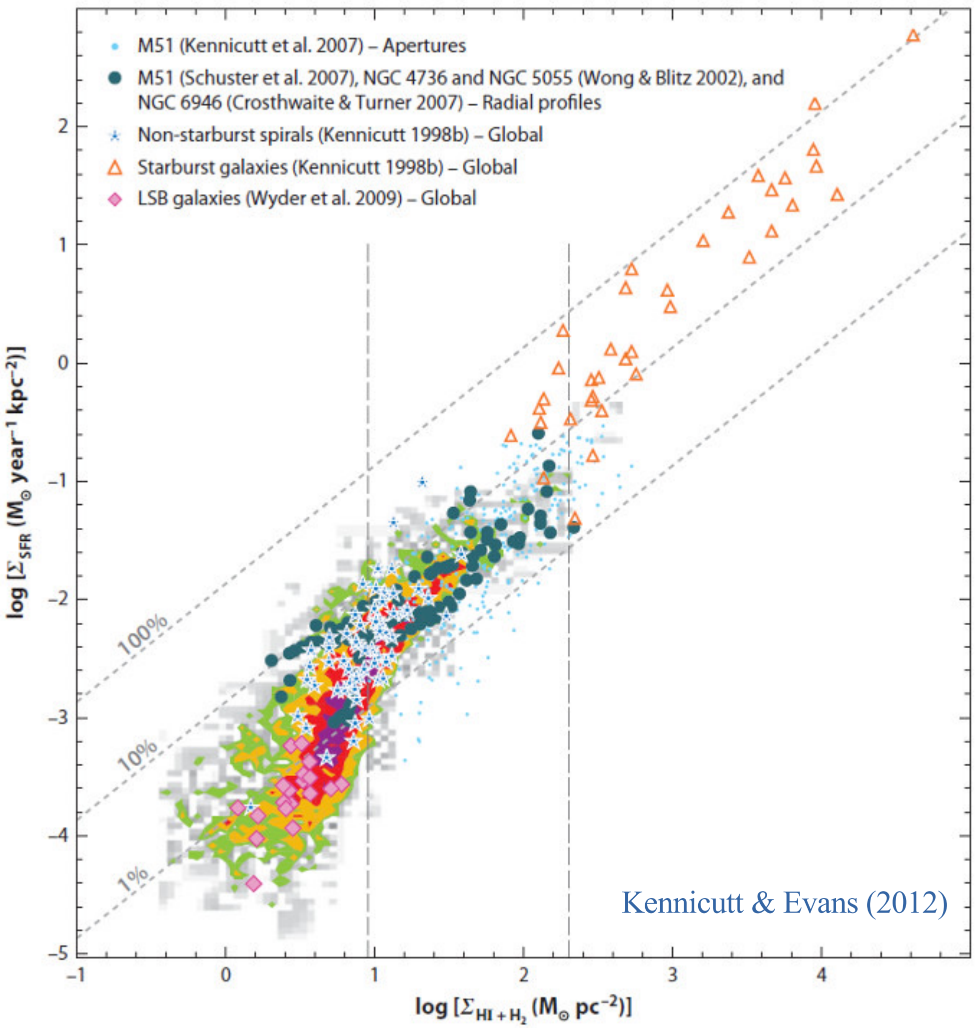

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Explain 1: free-fall timescale

Star-formation volume density

$$\rho_{SFR} \propto \frac{\rho_{gas}}{t_{ff}}$$

$$\propto \frac{\rho_{gas}}{\left(G\rho_{gas}\right)^{-0.5}} \propto \rho_{gas}{}^{1.5}$$

Assuming constant scale-height

$$\Sigma_{SFR} \propto \rho_{SFR}$$

$$\Sigma_{gas} \propto \rho_{gas}$$

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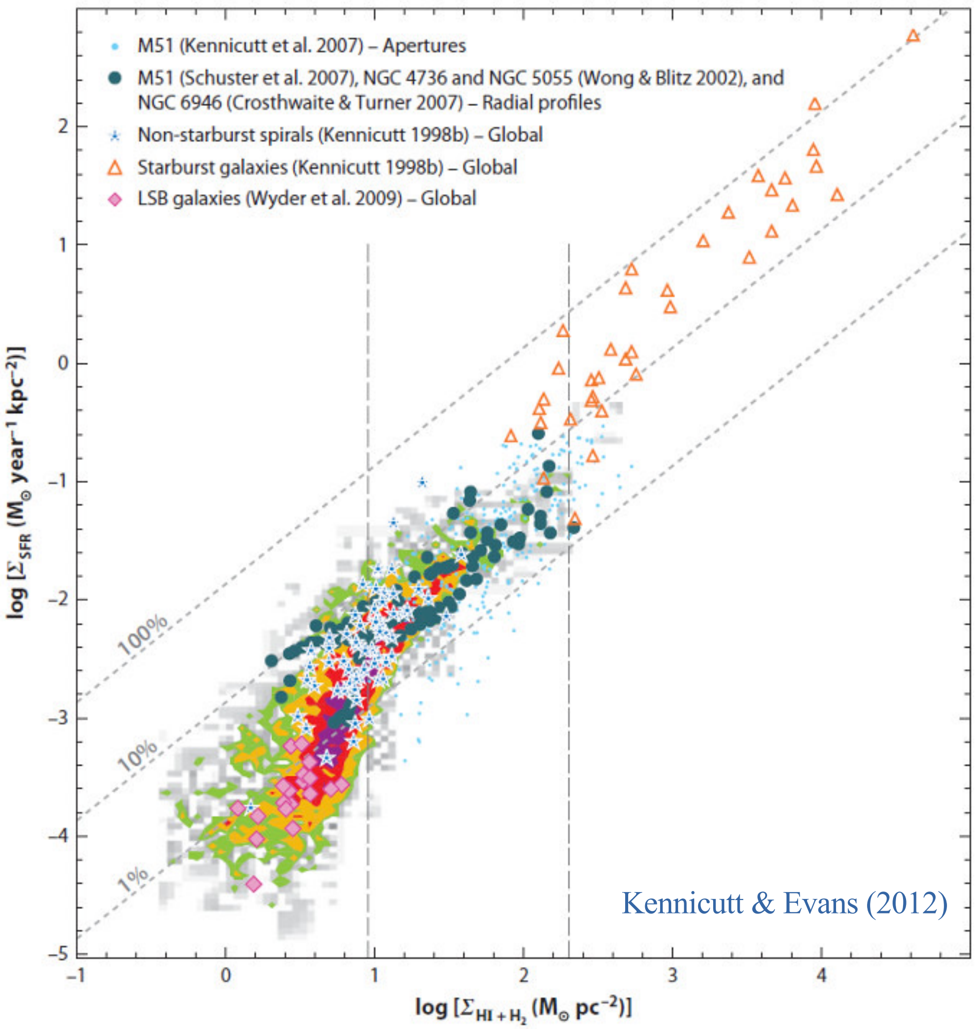

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Explain 2: galactic dynamical timescale

$$\Sigma_{SFR} \propto \frac{\Sigma_{gas}}{\tau_{dyn}} \propto \Sigma_{gas}\Omega_{gas}$$

Ω_{gas} : local orbital timescale

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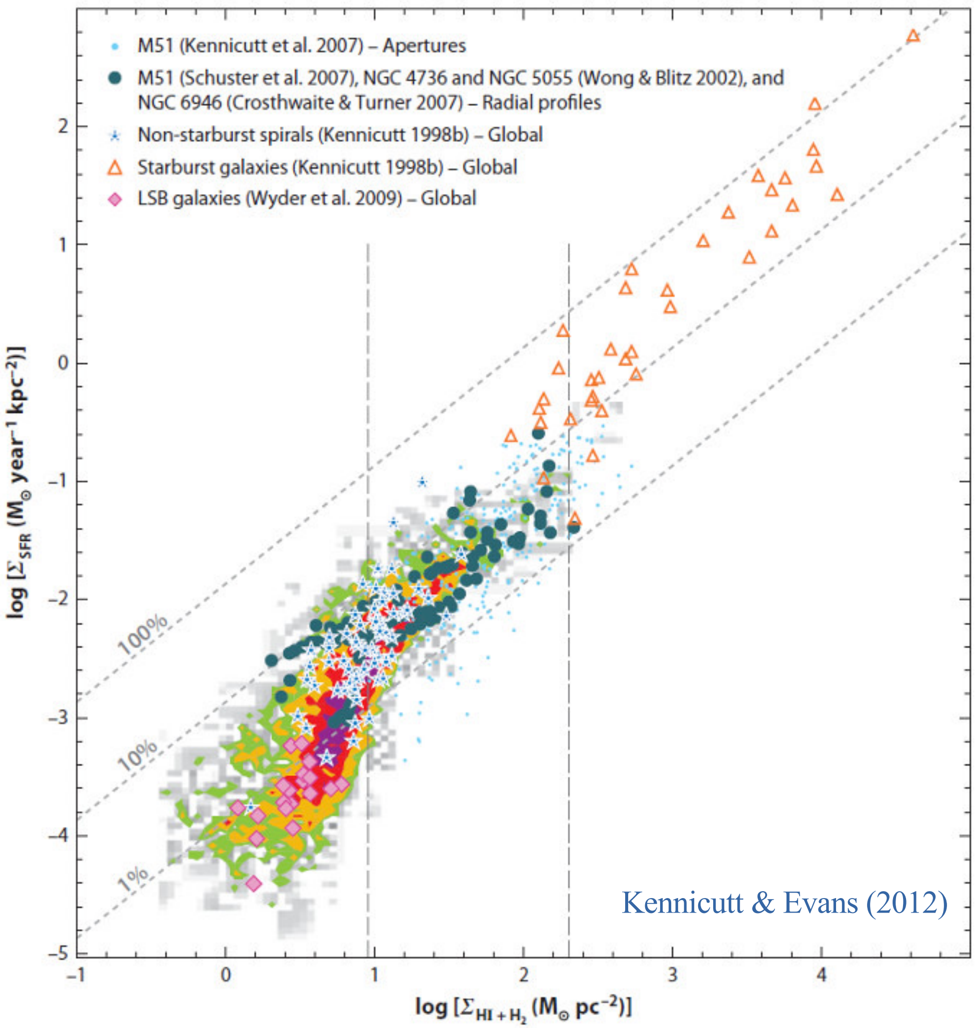

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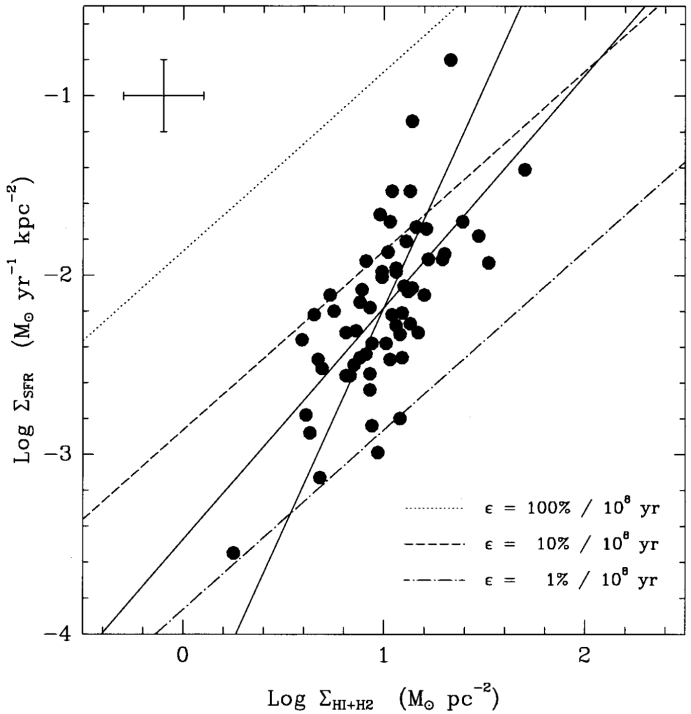
KS-law Implies a very low SFE (not well explained by far)
There are some other physics to quench Star-formation

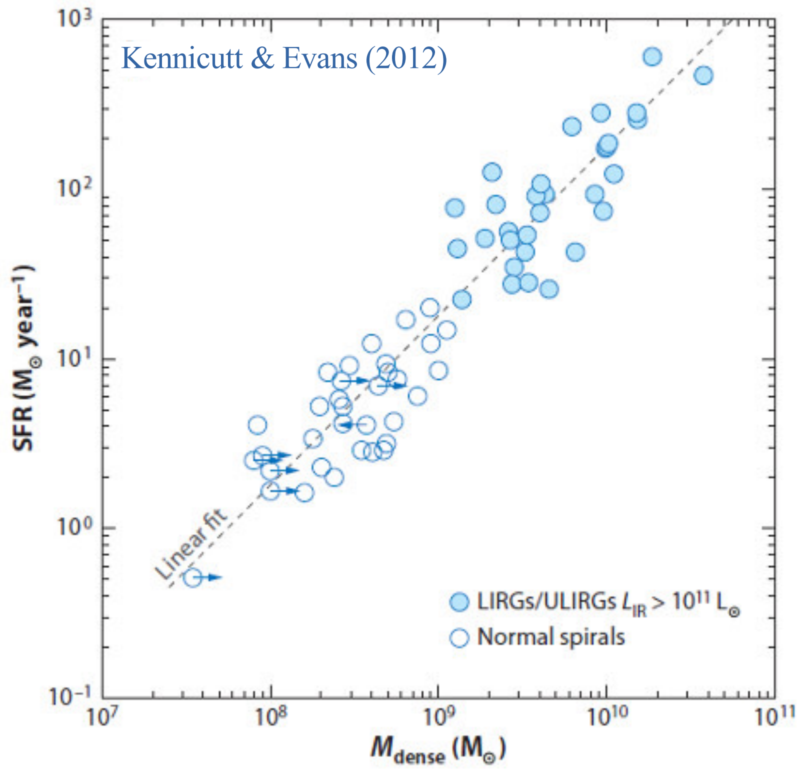


Uncertain Baron Physics

(1) Halo Occupation Distribution (2) Kennicutt-Schmidt Law (3) **Gao-Solomon Relation**

Gao & Solomon 2004, ApJ, 606, 271

$$SFR = (1.8\times10^{-8})\times\left(\frac{M_{dense}}{1\ M_{\odot}}\right) M_{\odot} yr^{-1}$$



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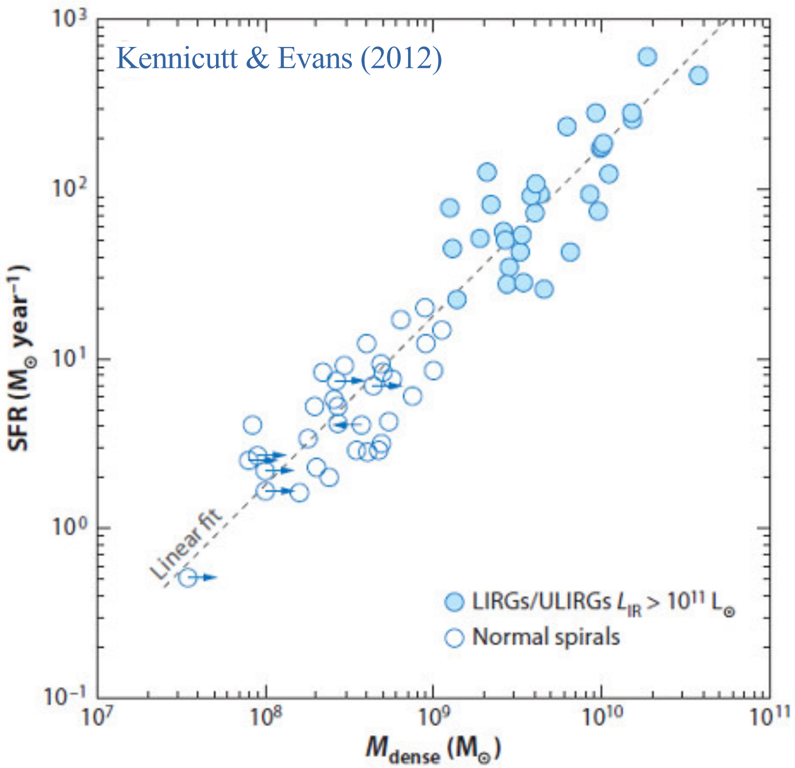
Again this law implies very low star-forming efficiency.
In addition, the definition of dense gas remains ambiguous (Jiao et al. submitted).



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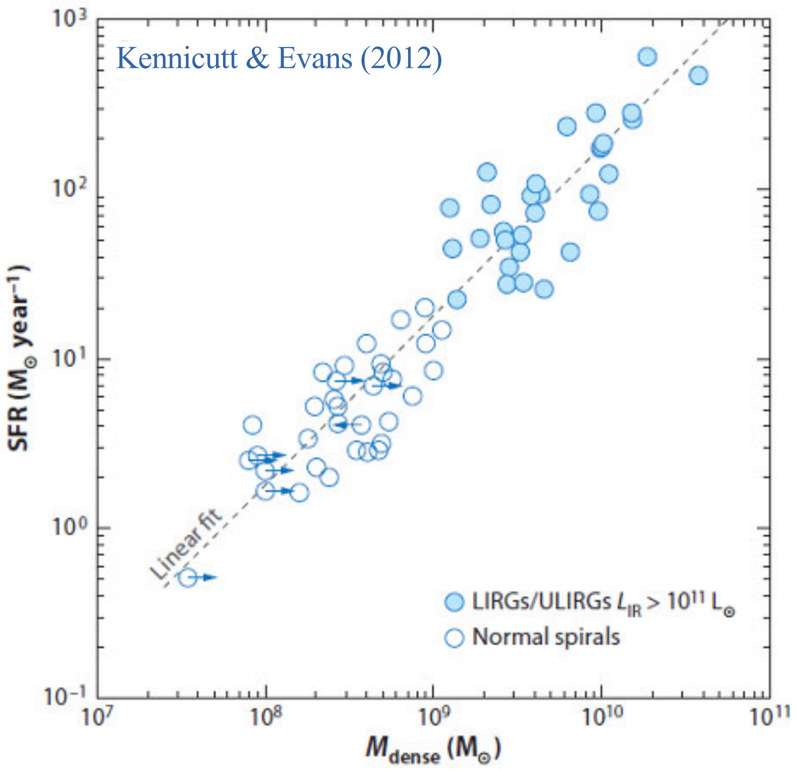
$$SFR = (1.8\times10^{-8})\times\left(\frac{M_{dense}}{1\ M_\odot}\right) M_\odot yr^{-1}$$

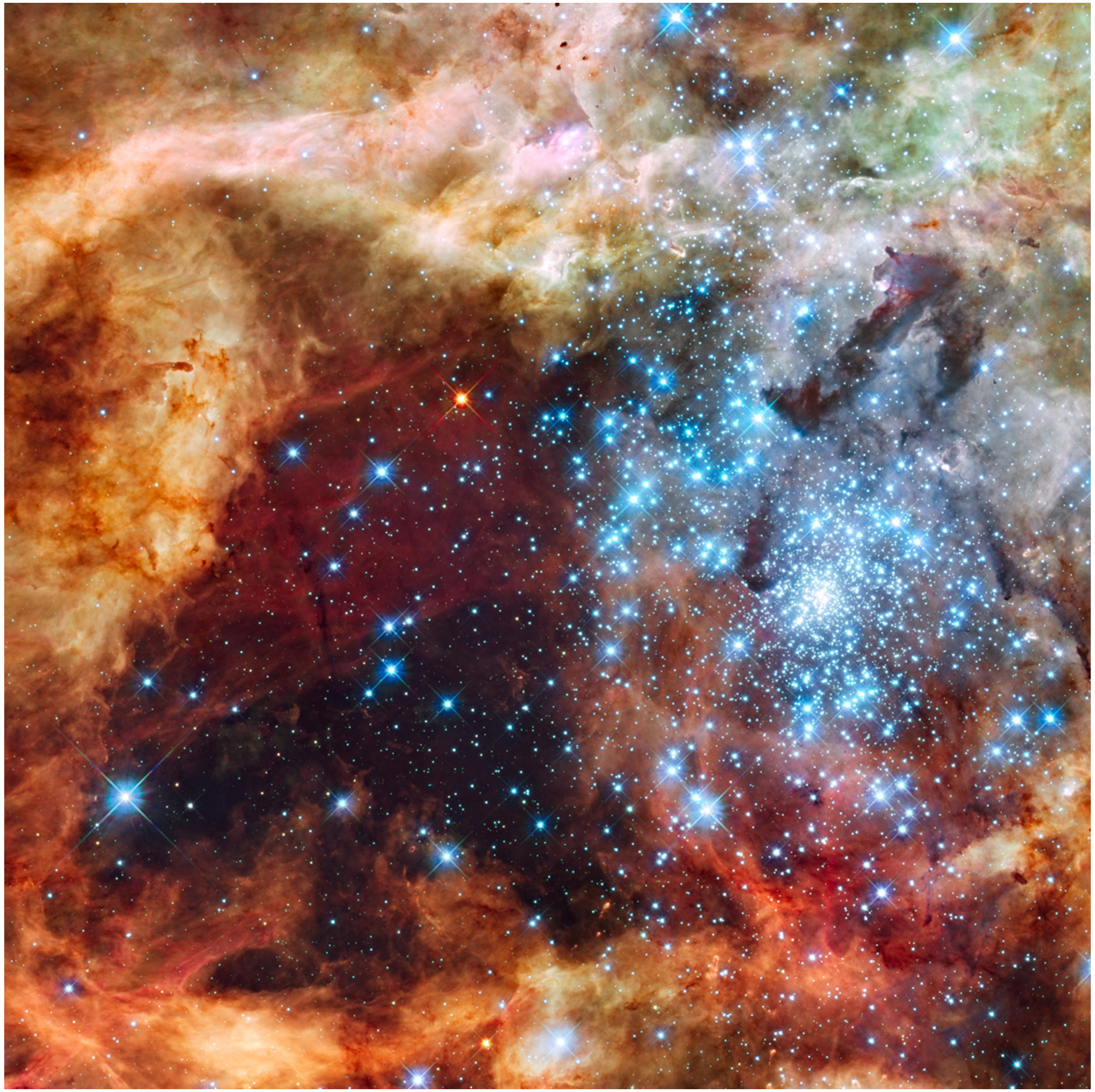
This Law is very strange.

Ten molecular clouds of $10^5\ M_\odot$ of gas mass and one molecular cloud with $10^6\ M_\odot$ of gas mass consume the same amount of gas mass to star-formation every year, in spite that the stars they form are very different

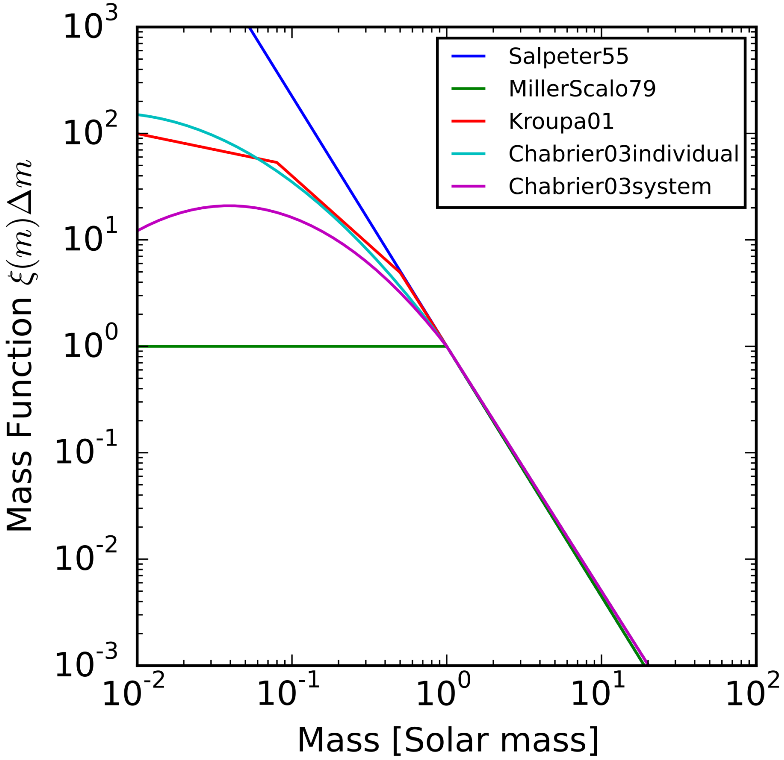
(Jiao, Xu, Liu et al. in prep.)

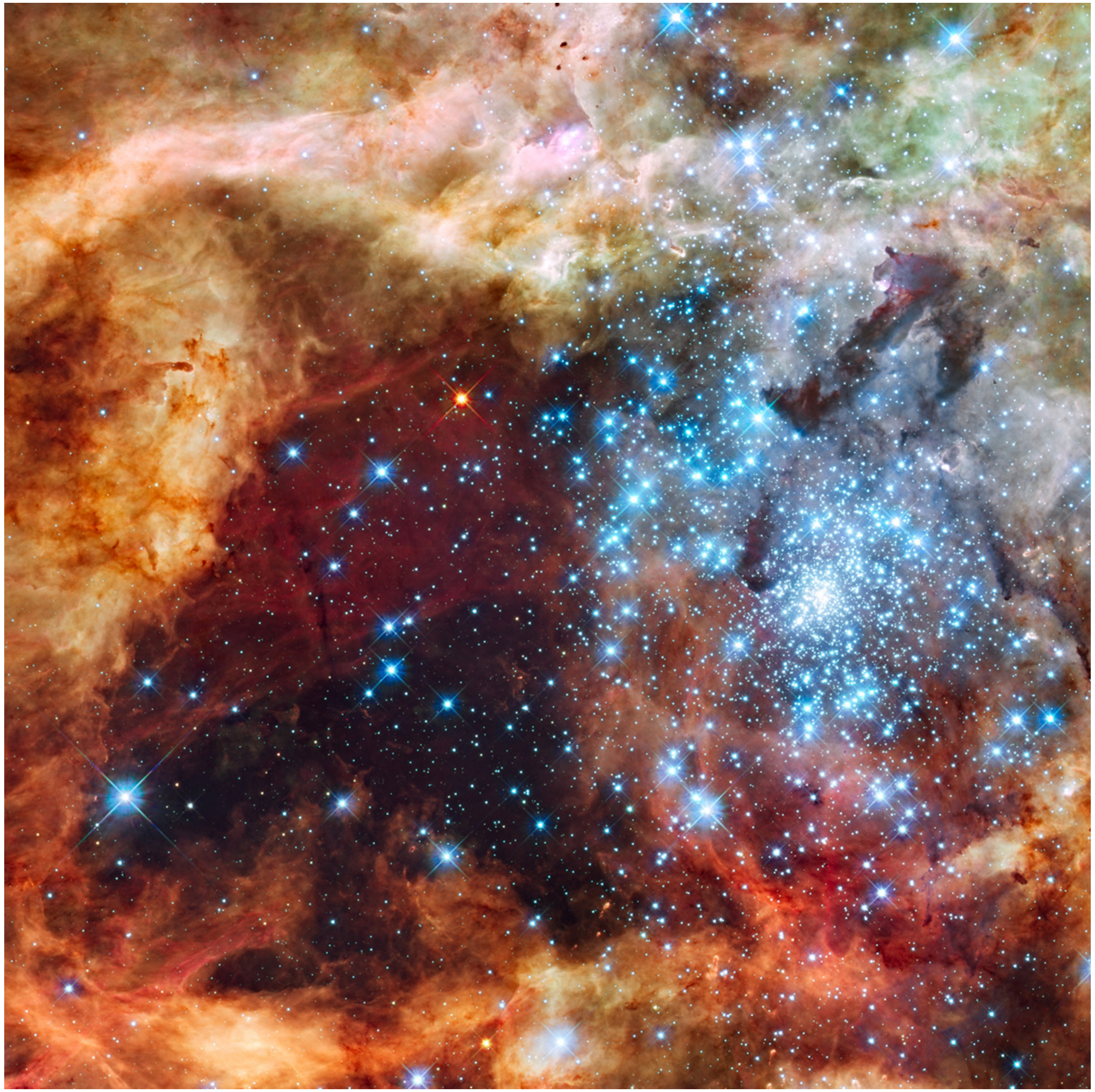


Stars Form in Clusters (Multiplicity is Essential)



Stellar Initial Mass Function

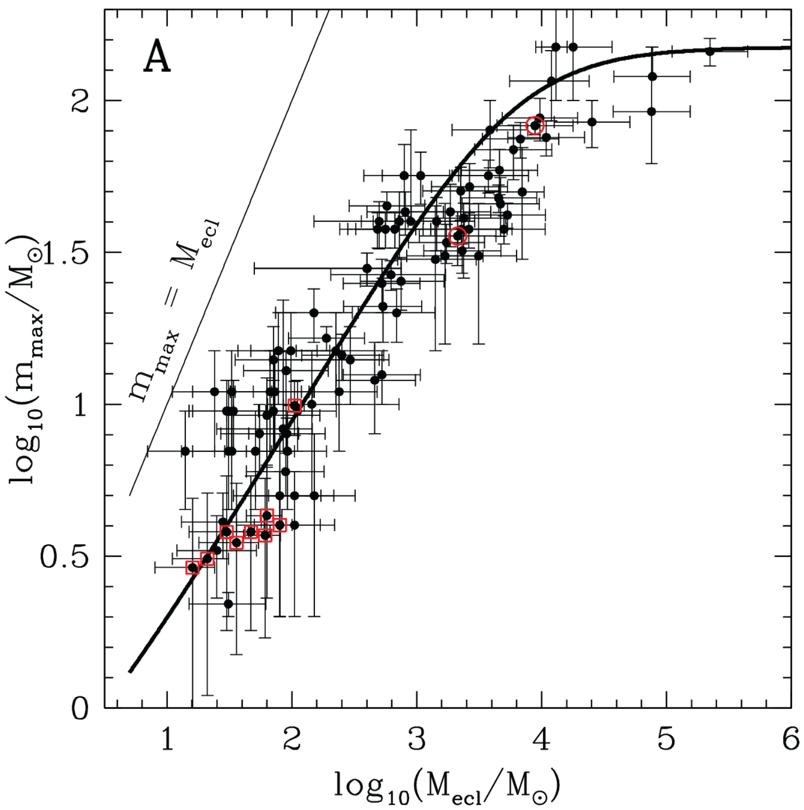


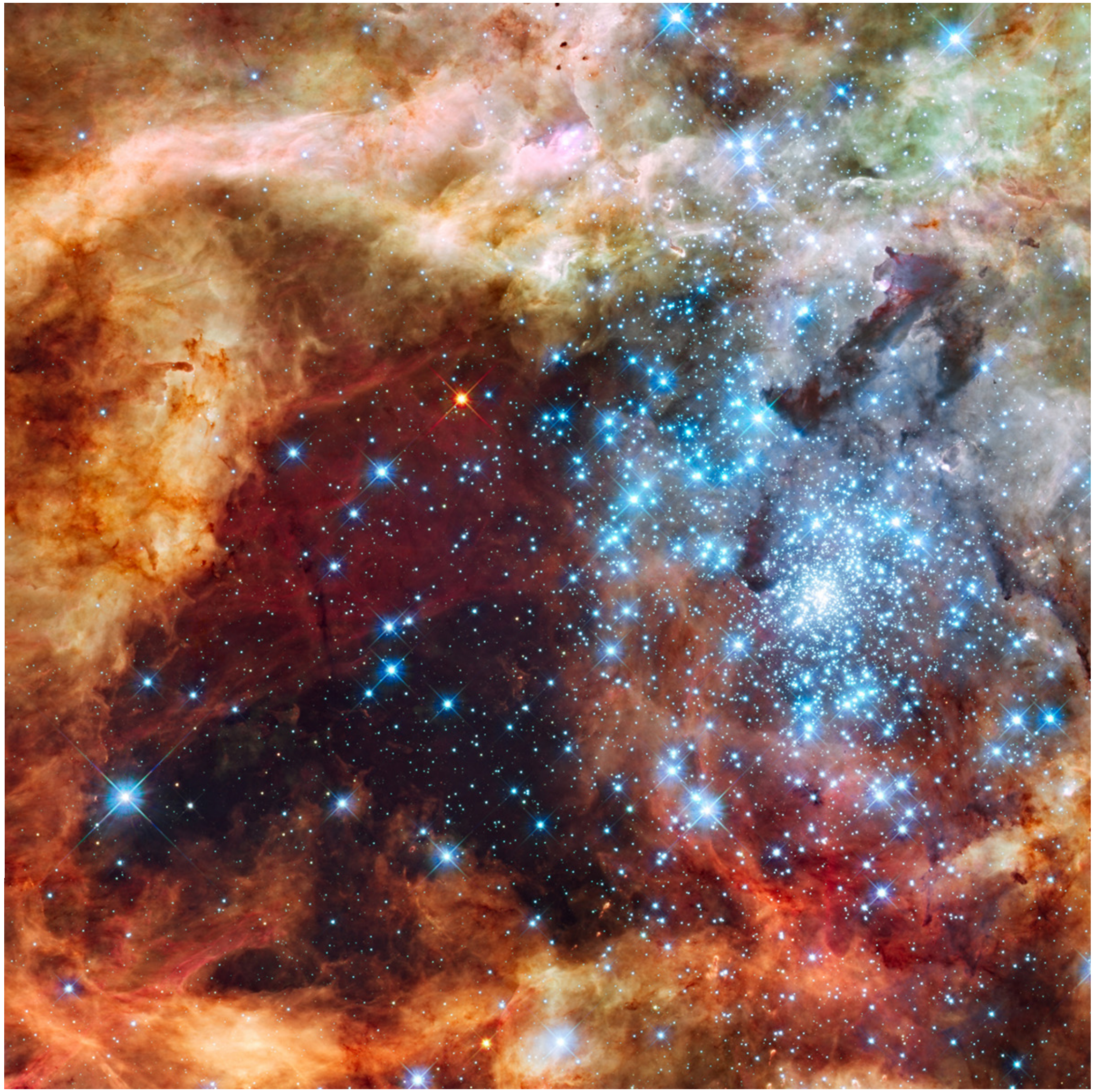


https://en.wikipedia.org/wiki/Initial_mass_function

$M_{max} - M_{ecl}$ relation

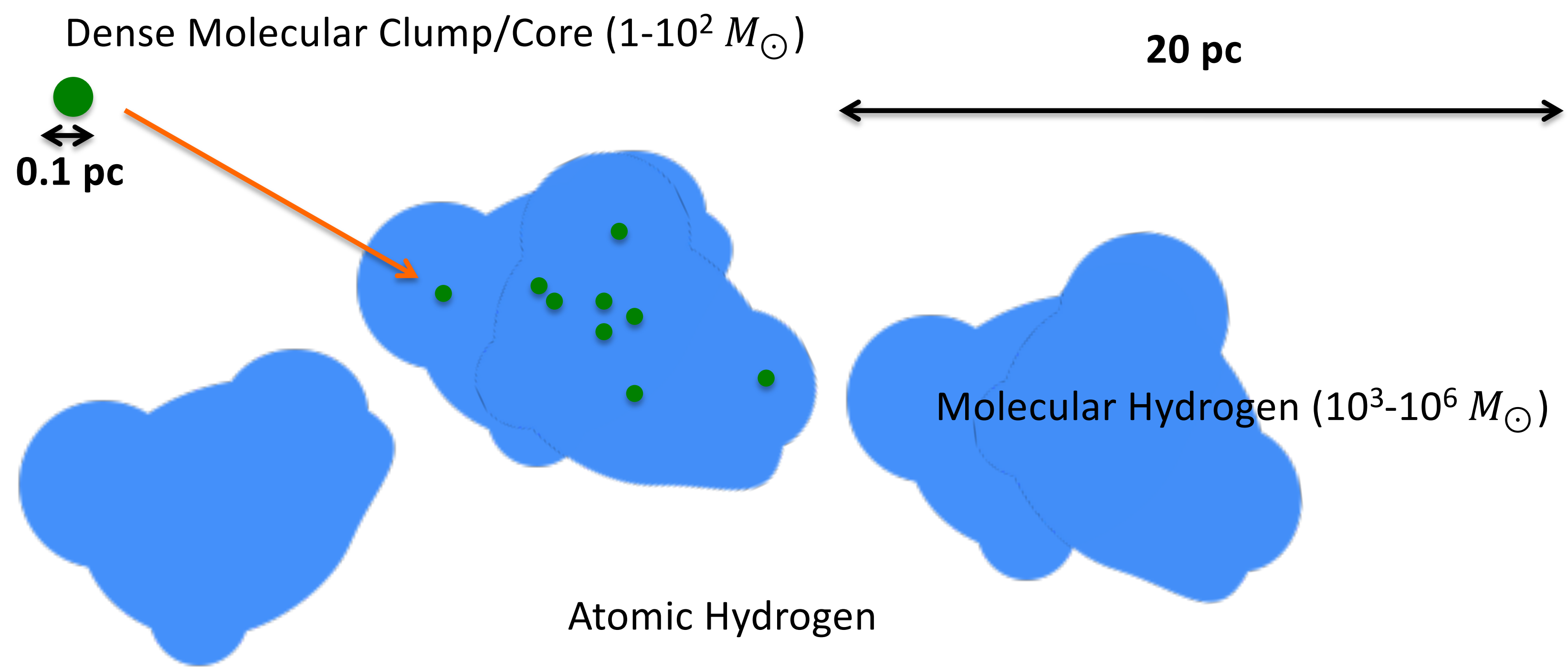
Weidner, Kroupa, Bonnell 2010,MNRAS, 401, 275





A Simplified Picture of Interstellar Medium

1 pc = 3×10^{16} meters



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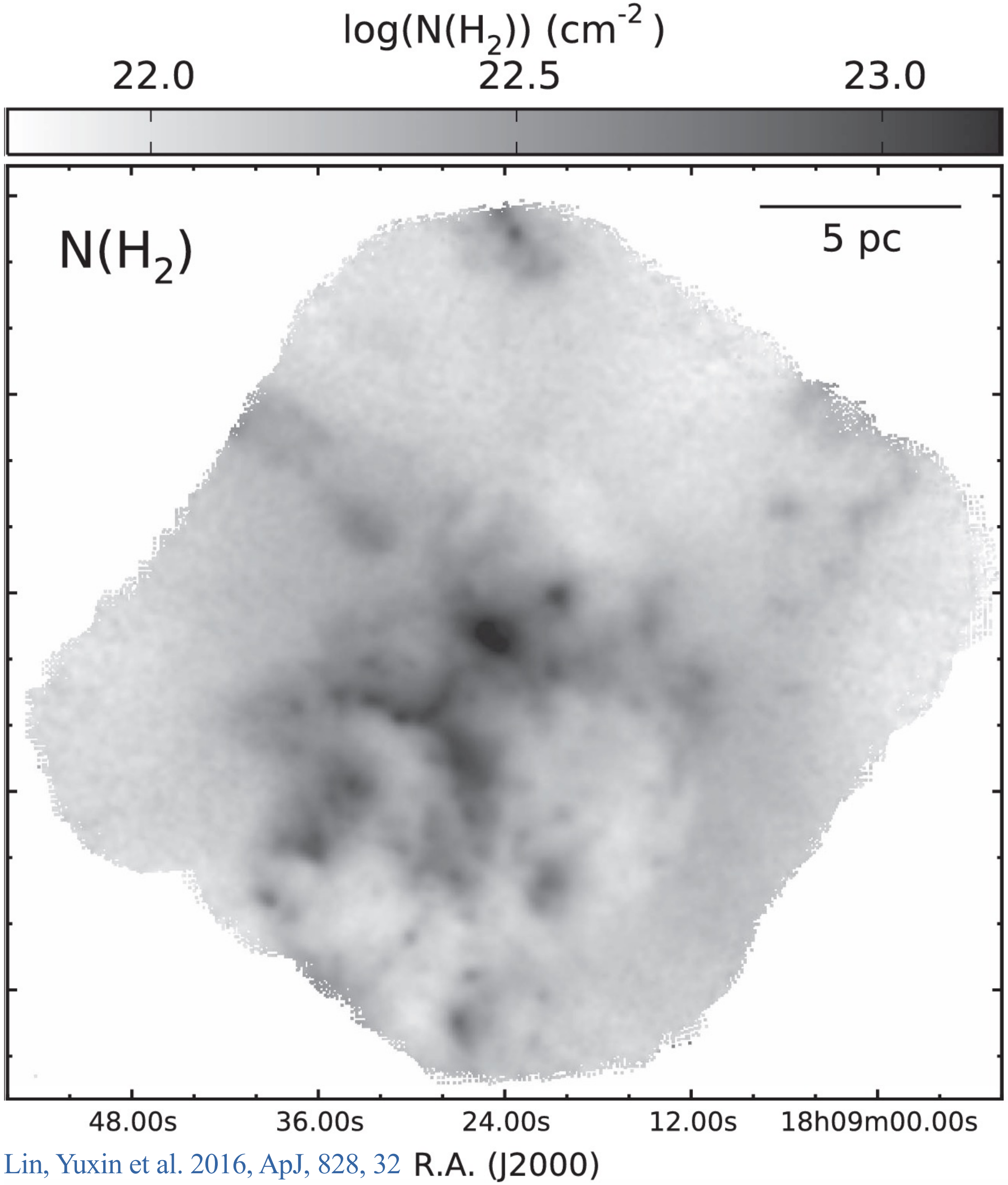

Lin, Yuxin et al. 2016, ApJ, 828, 32

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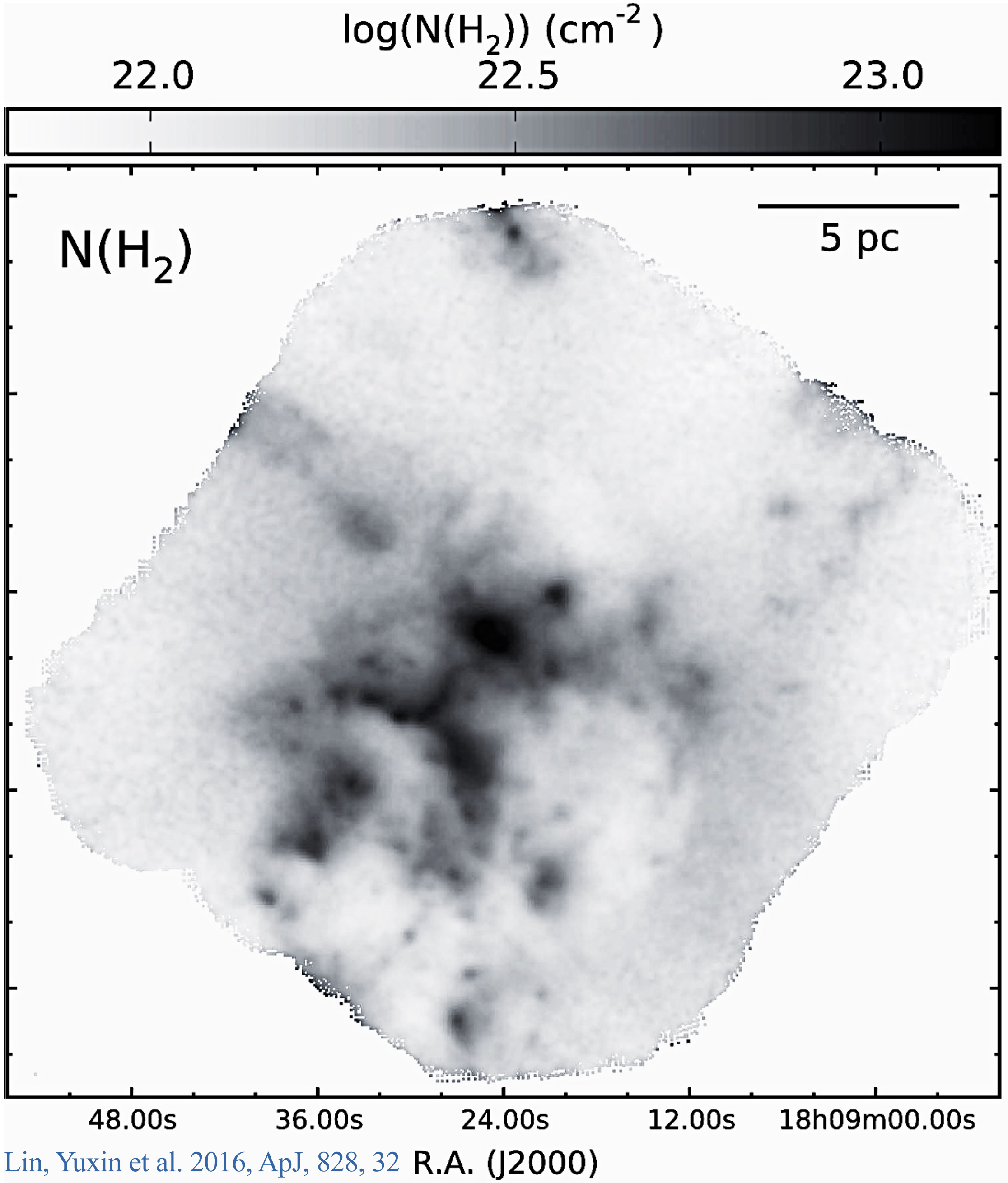

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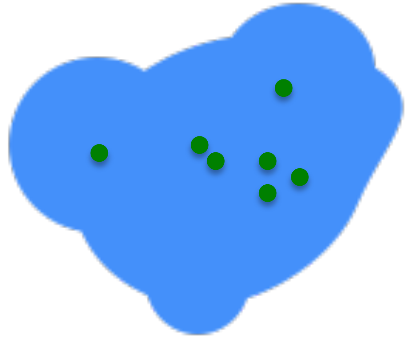
Criterion for Self-Gravitational Fragmentation



Equation of Continuity:

<u>mass:</u>

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

<u>momentum:</u>

$$\frac{\partial (\rho v_i)}{\partial t} \hat{\imath} + \vec{\nabla} \cdot \big((\rho v_i)\vec{v}\big)\hat{\imath} = 0$$

Momentum density $p_i = \rho v_i$

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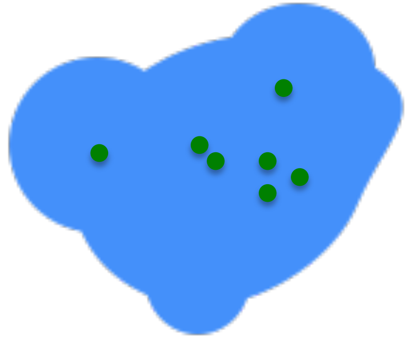

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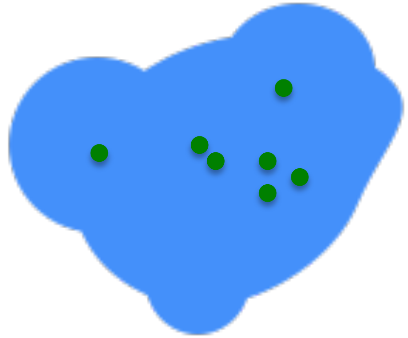

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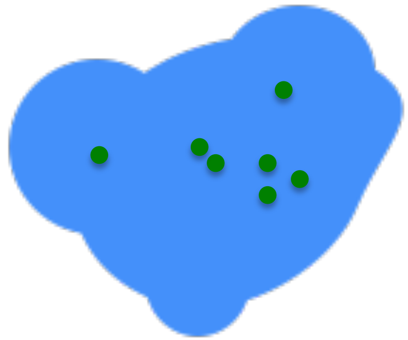

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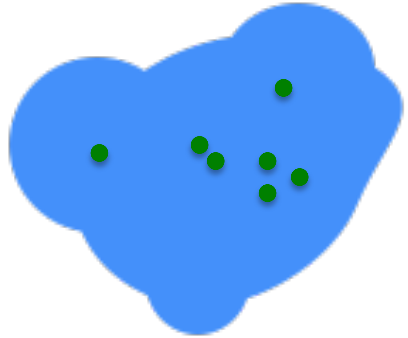

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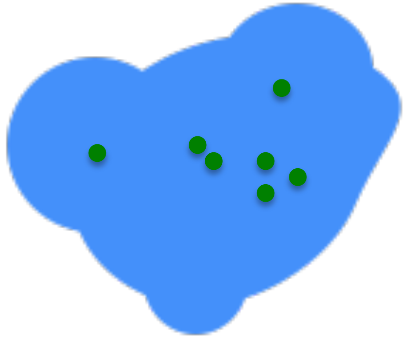

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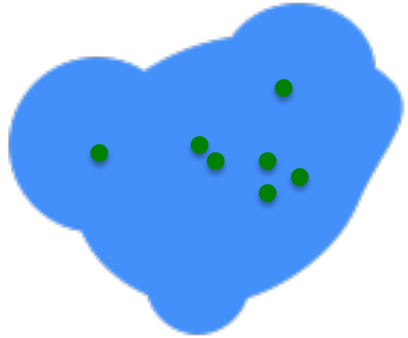

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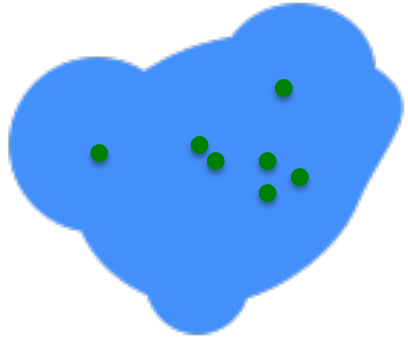

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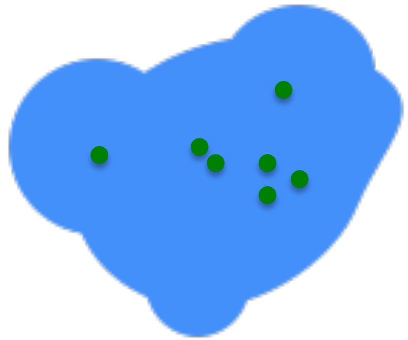

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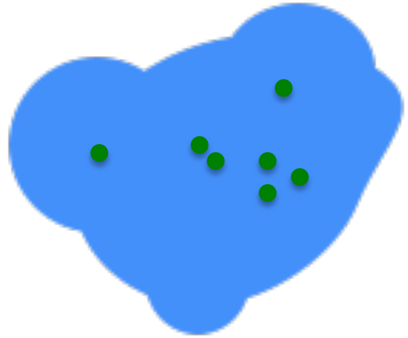

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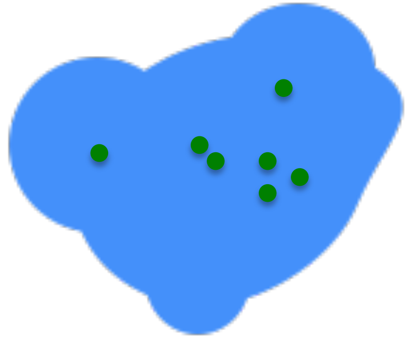

Equation of Continuity (with pressure *P* and gravitational acceleration $\vec{g}$):

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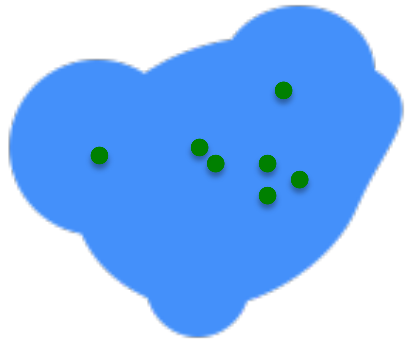
Criterion for Self-Gravitational Fragmentation



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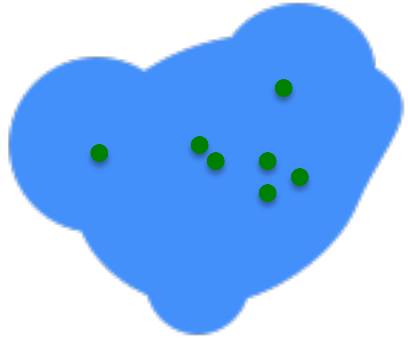
$$\frac{\partial (\rho v_i)}{\partial t} \hat{\imath} + \vec{\nabla} \cdot ((\rho v_i) \vec{v}) \hat{\imath} = \left[-(\vec{\nabla} P)_i - \rho (\vec{\nabla} \phi)_i \right] \hat{\imath}$$

Poisson Equation

$$\nabla^2 \phi = 4 \pi G \rho$$

Criterion for Self-Gravitational Fragmentation



Equation of Continuity (with pressure *P* and gravitational acceleration $\vec{g}$):

mass:

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momentum:

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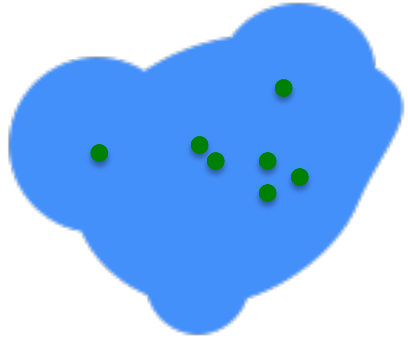

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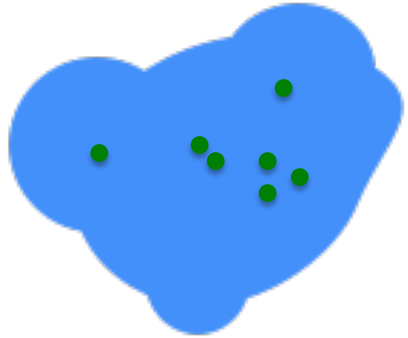

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Poisson Equation

$$\nabla^2 \phi = 4\pi G \rho$$

Equation of state

$\mathrm{P} = {c_s}^2 \rho,$

$c_s \equiv$isothermal sound speed

Perturbation theory

$\rho = \rho_0 + \rho_1, \quad \rho_0 \gg \rho_1, \quad \vec{\nabla} \rho_0 = 0$ [uniform initial condition]

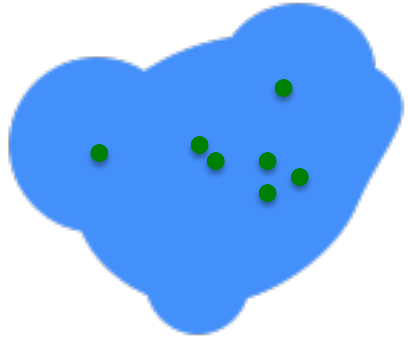
$\mathrm{P} = P_0 + P_1, \quad P_0 \gg P_1, \quad \vec{\nabla} P_0 = 0$ [uniform initial condition]

$\vec{v} = \vec{v}_0 + \vec{v}_1, \quad \vec{v}_0 = 0$ [initially quiescent cloud]

$\phi = \phi_0 + \phi_1, \quad \phi_0 \gg \phi_1$

Plugging into the two equations of continuity, and the Poisson equation

Criterion for Self-Gravitational Fragmentation



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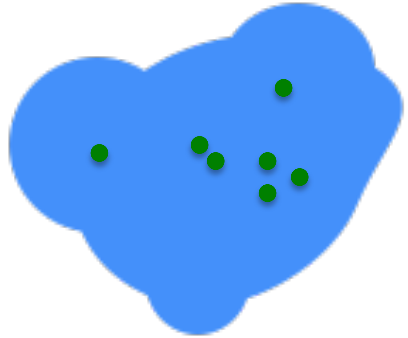

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$$\frac{\partial}{\partial t}(1) \Longrightarrow \frac{\partial^2 \rho_1}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \frac{\partial \overrightarrow{v_1}}{\partial t} = 0$$

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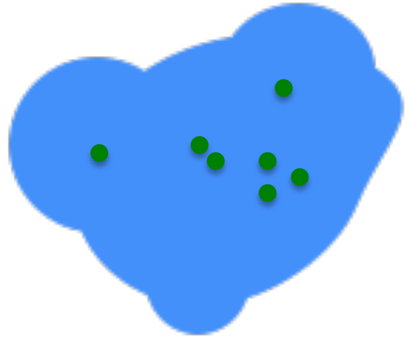

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Dispersion relation $\quad \frac{\partial^2 \rho_1}{\partial t^2} - {c_s}^2 \nabla^2 \rho_1 = 4\pi G \rho_0 \rho_1$

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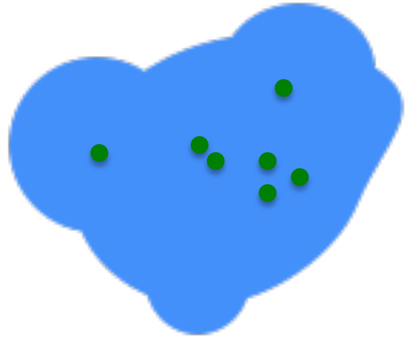

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$$\Longrightarrow -\omega^2 + k^2 {c_s}^2 = 4\pi G \rho_0$$

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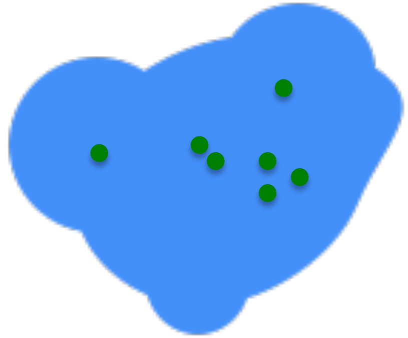

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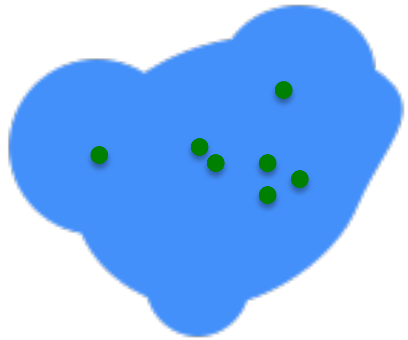

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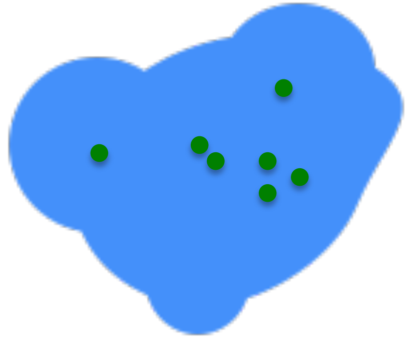

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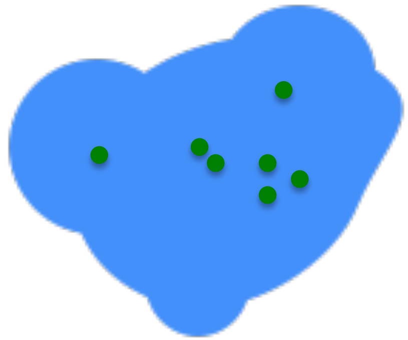

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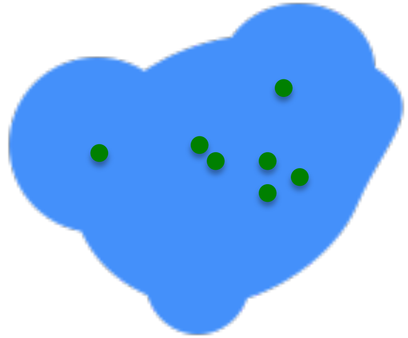

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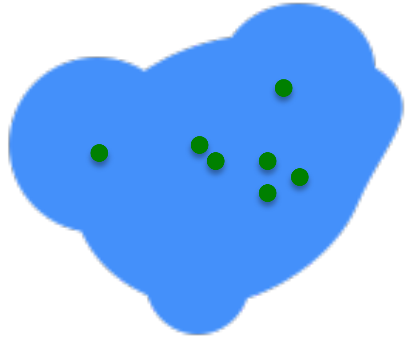

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$M_J \equiv \frac{4}{3}\pi\left(\frac{\lambda_J}{2}\right)^3.$ **(Jeans mass)**

An initially uniform and quiescent molecular cloud will fragment into sub-structures of Jeans masses. The spatial separations of these sub-structures are approximately the Jeans length.

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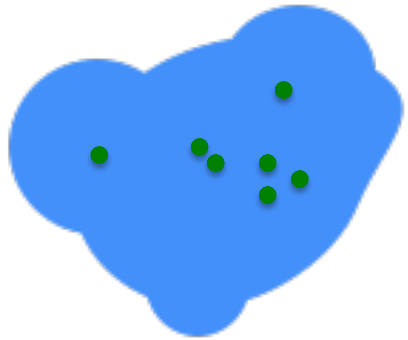

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An initially uniform and quiescent molecular cloud will just collapse to form a black hole.

What was the problem?

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(1) Linearization works OK with the 1st order equations, but does not make sense at the 0th order

$$\begin{cases} \rho = \rho_0 + \rho_1, \quad \rho_0 \gg \rho_1, \quad \vec{\nabla}\rho_0 = 0 \text{ [uniform initial condition]} \\ \mathrm{P} = P_0 + P_1, \quad P_0 \gg P_1, \quad \vec{\nabla}P_0 = 0 \text{ [uniform initial condition]} \\ \vec{v} = \vec{v}_0 + \vec{v}_1, \quad \vec{v}_0 = 0 \text{ [initially quiescent cloud]} \\ \phi = \phi_0 + \phi_1, \quad \phi_0 \gg \phi_1 \end{cases}$$

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0th order momentum eq.

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$$\frac{\partial(v_i)}{\partial t} + \vec{v} \cdot (\vec{\nabla} v_i) = -\frac{(\vec{\nabla}P)_i}{\rho} - (\vec{\nabla}\phi)_i$$ 0th order momentum eq.

$$\Longrightarrow (\vec{\nabla}P_0)_i = 0 = \rho_0(\vec{\nabla}\phi_0)_i$$

$$\Longrightarrow \vec{\nabla}\phi_0 = 0 \Longrightarrow \nabla^2\phi = 0 \neq 4\pi G\rho_0$$

An initially quiescent cloud cannot exist. A finite-sized cloud will undergo gravitational contraction.

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An initially quiescent cloud cannot exist. A finite-sized cloud will undergo gravitational contraction.

(2) The growing timescale is minimized when $|\omega^2|$ is maximized at $k = 0$

Global collapse has a shorter characteristic timescale than perturbation growth

$$\omega^2 = k^2 {c_s}^2 - 4\pi G\rho_0$$

An initially uniform molecular cloud cannot fragment to form a cluster of star

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To make cloud fragmentation efficient, we need to make the global free-fall timescale larger than the local free-fall timescale

$$t_{ff} = \sqrt{\frac{3\pi}{16G\rho}}$$

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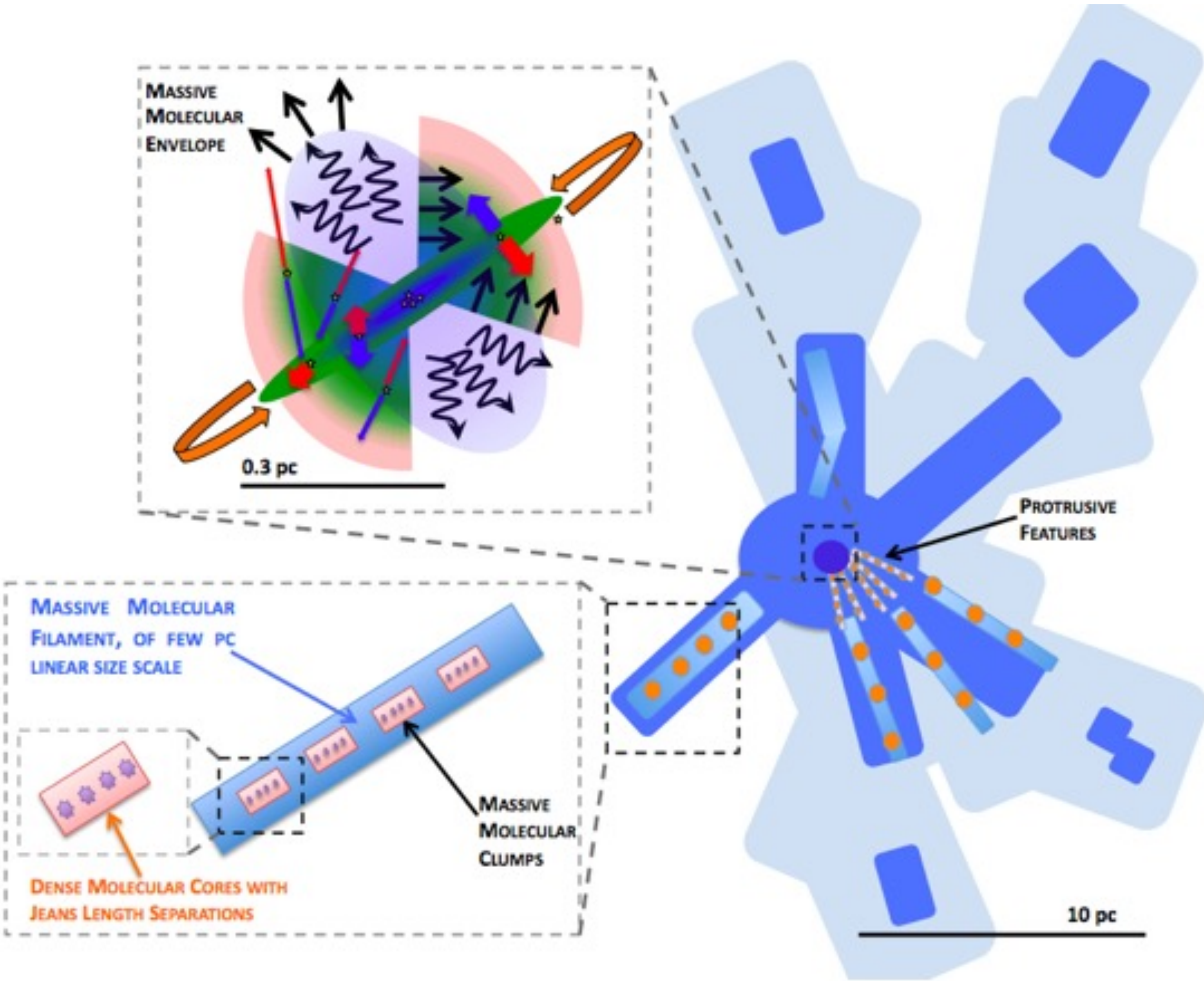
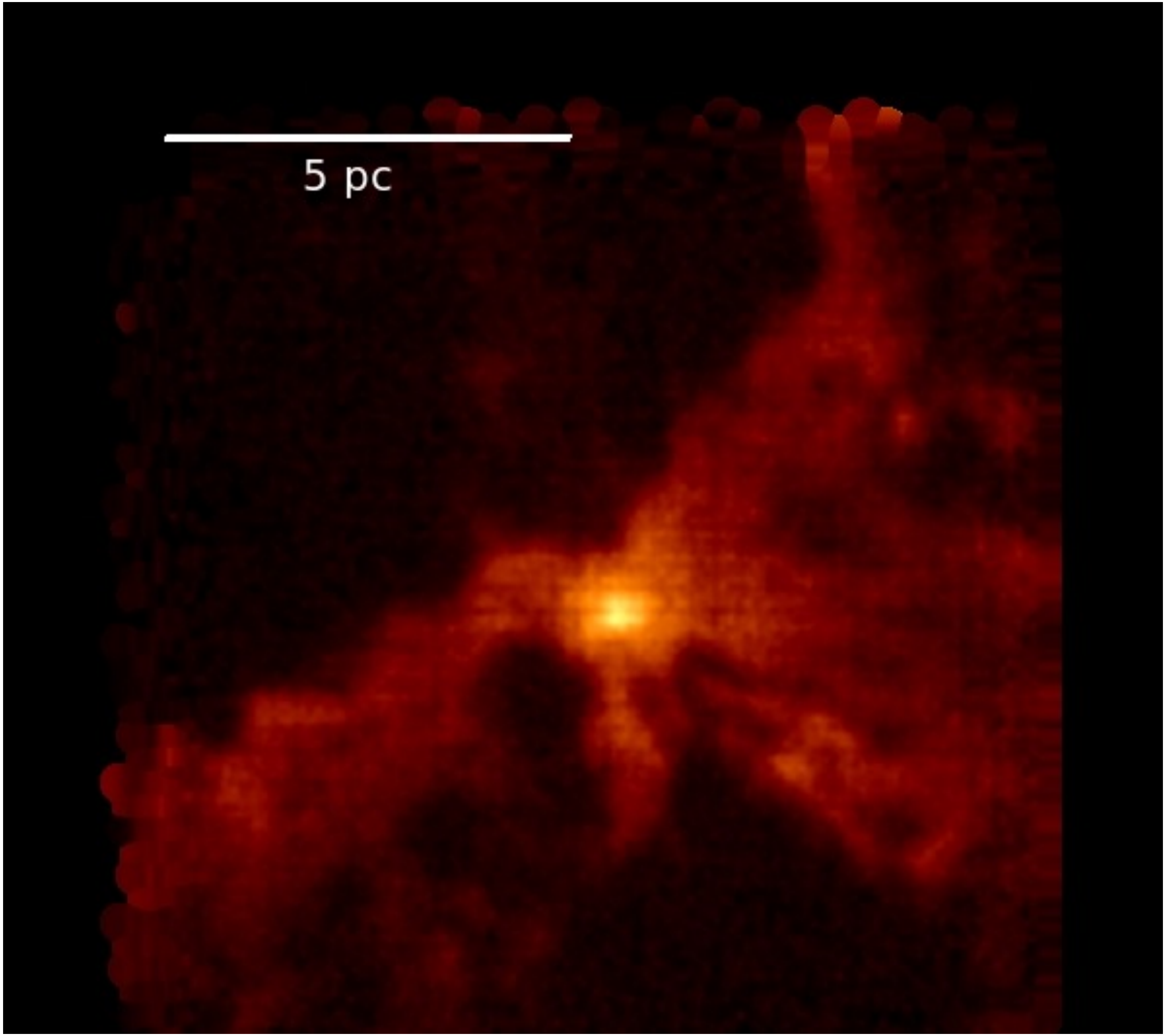
c.f. Larson, R. D. 1985, MNRAS, 214, 379

Molecular gas mass needs to be concentrated to sheets or filamentary structures

My Proposals



Liu, H. B. et al. 2012, ApJ, 745, 61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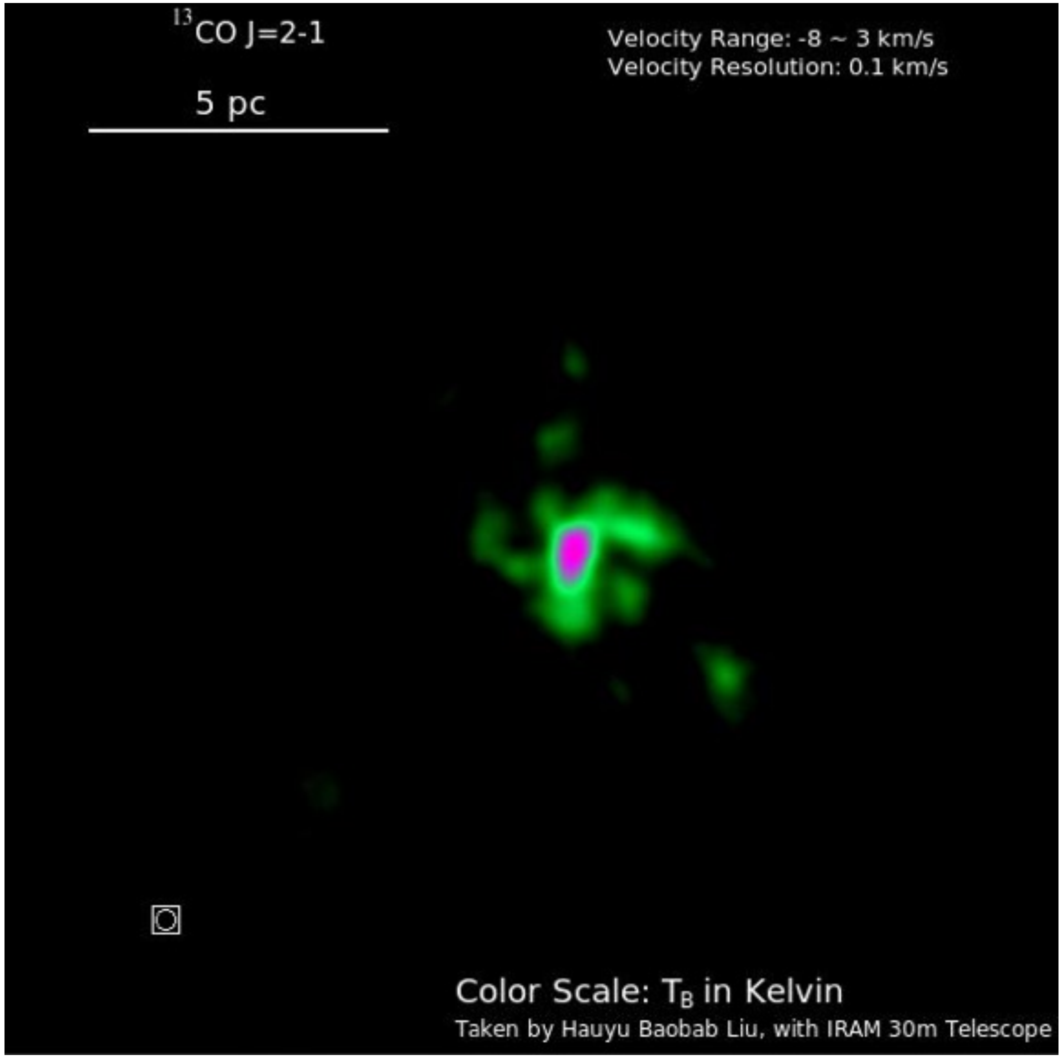
A Tomographic View in the ^{13}CO Velocity Channel Map



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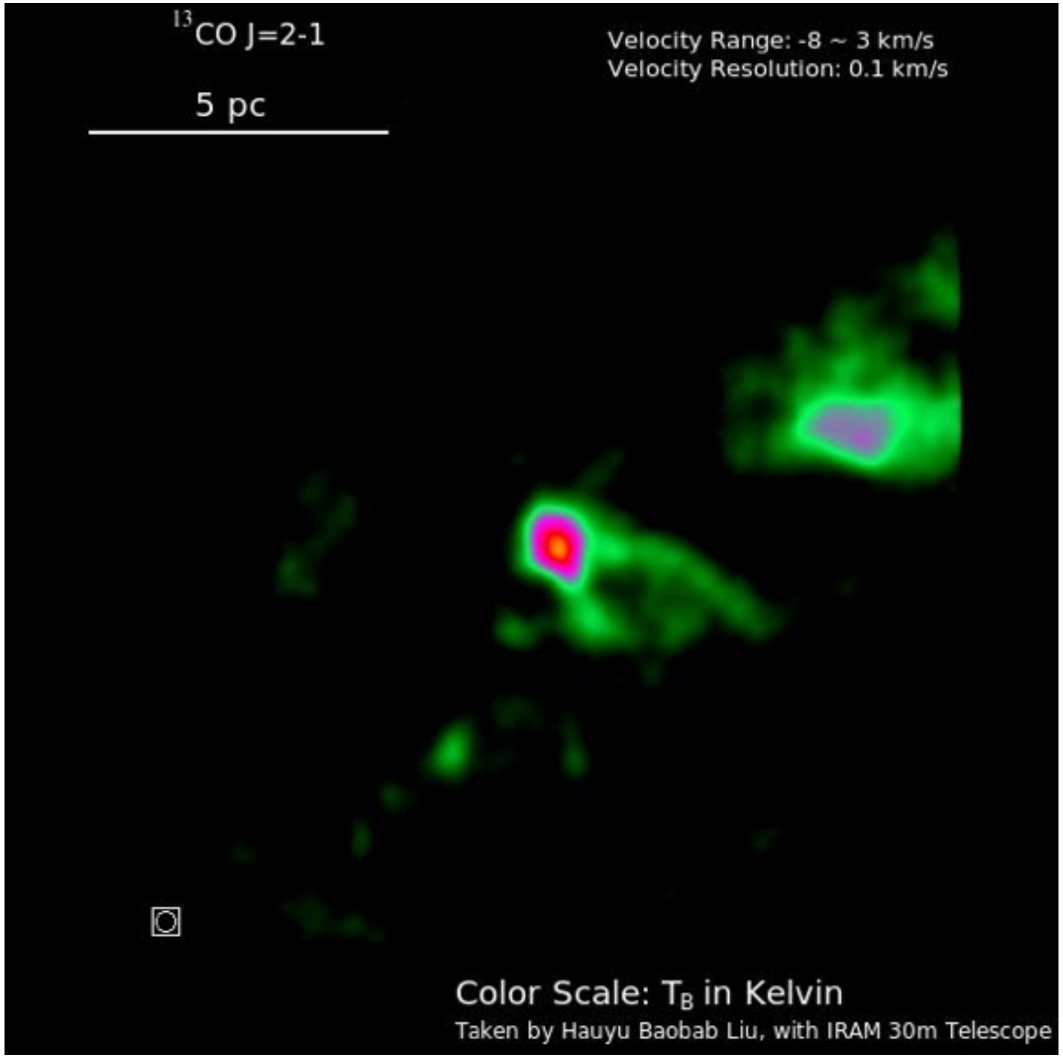
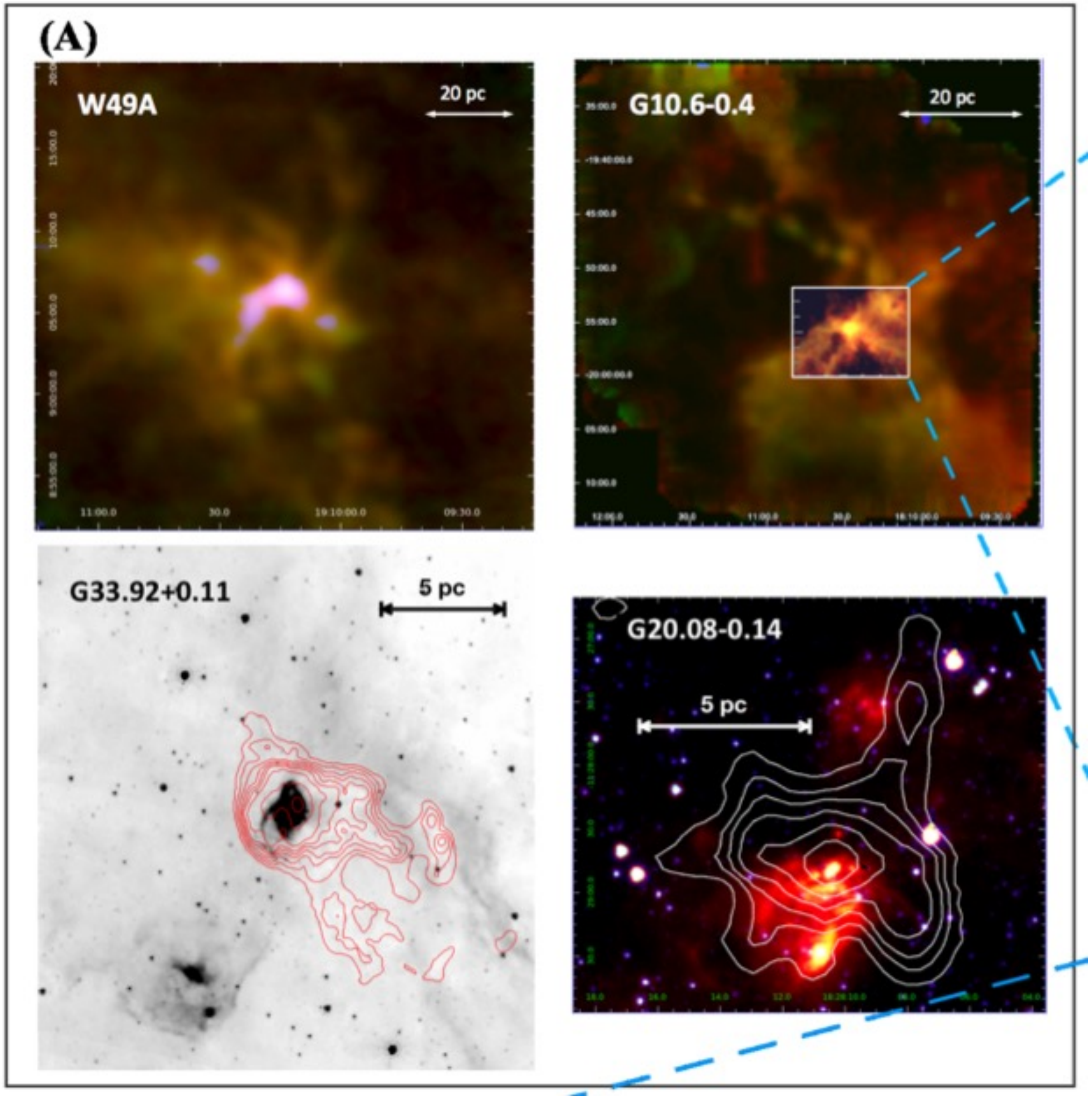
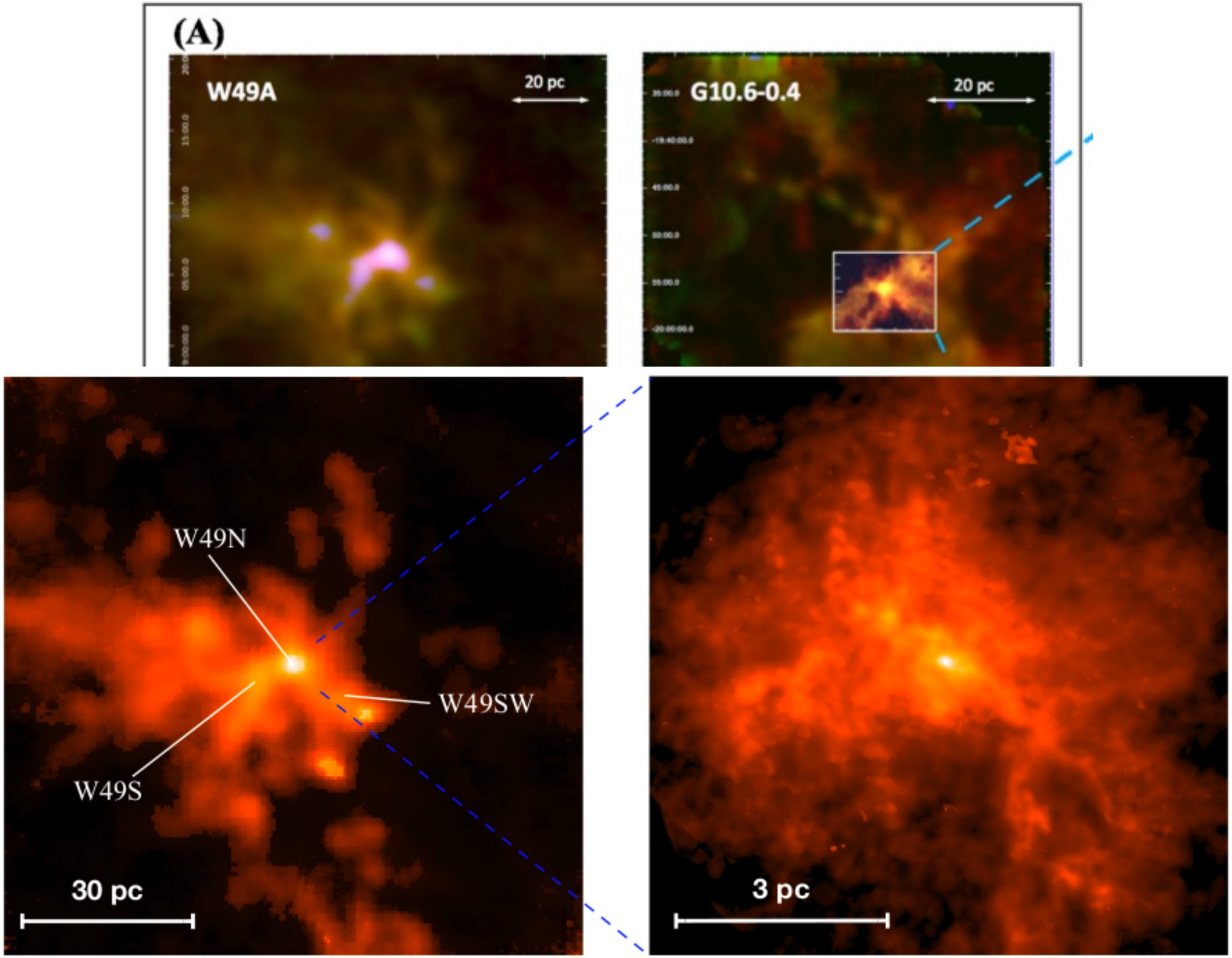


OB Cluster-forming Regions in Actual Observations



W49A



Galvan-Madrid, R.. et al. 2013, A&A, 779, 121

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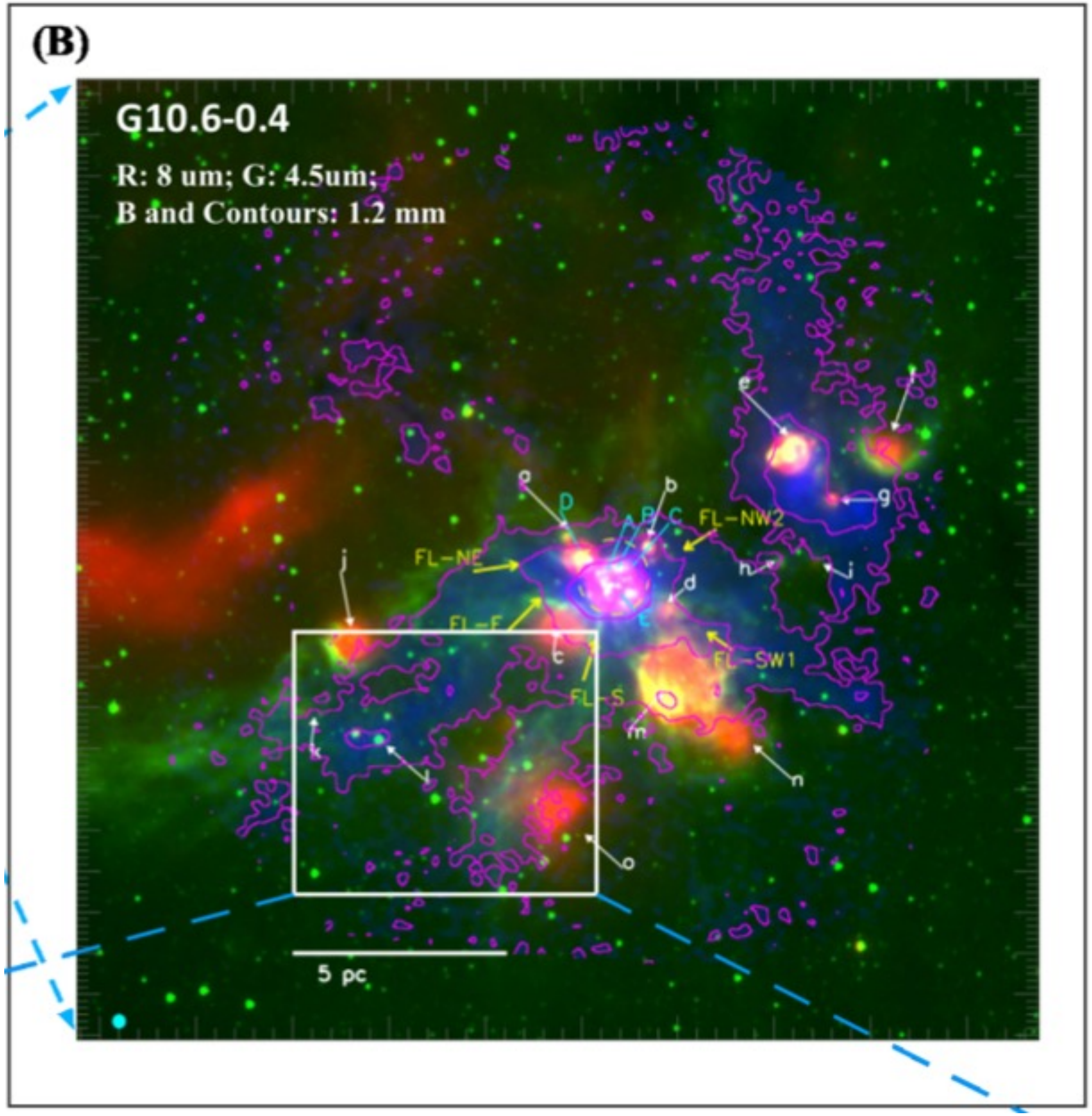

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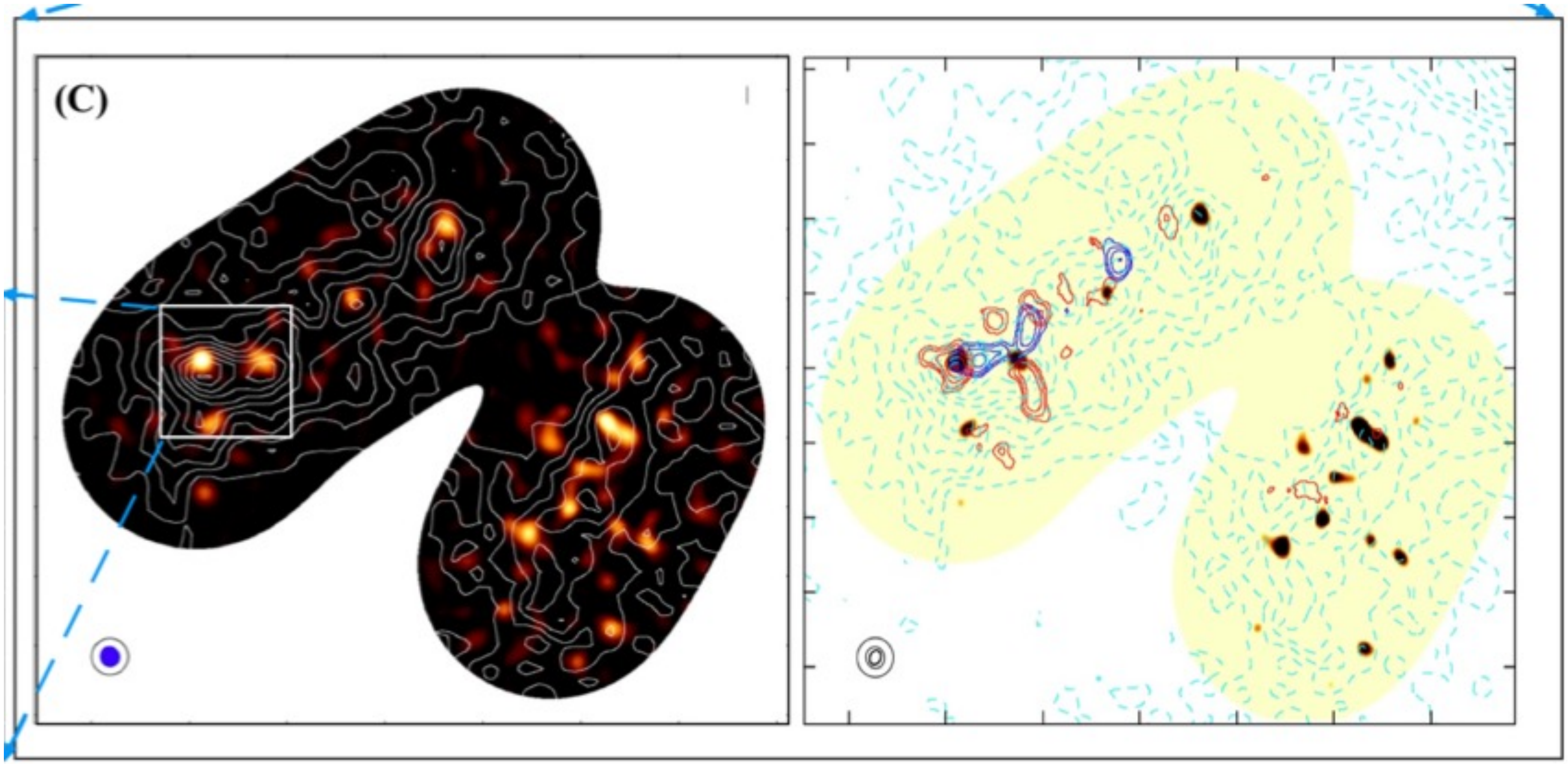

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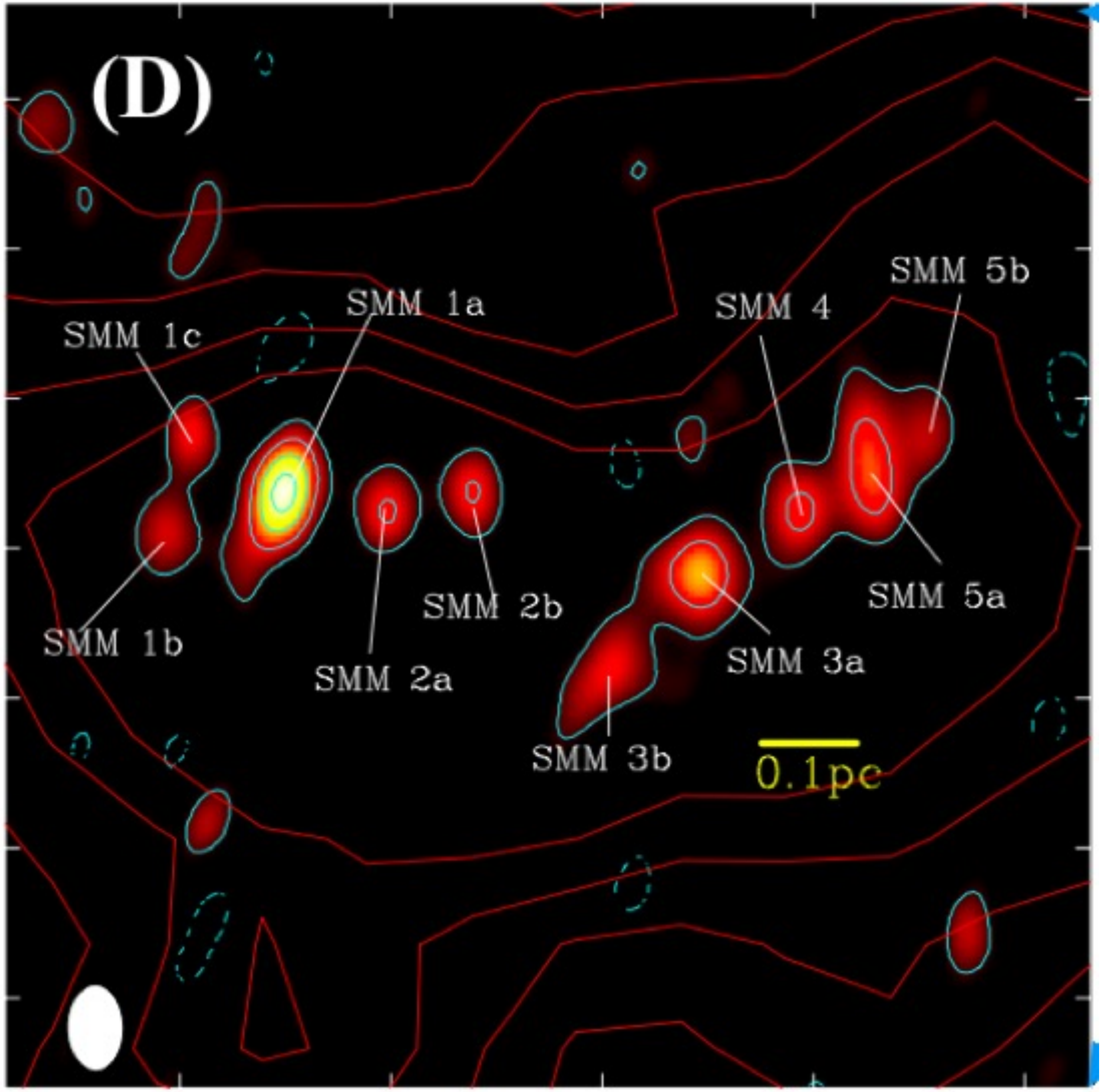
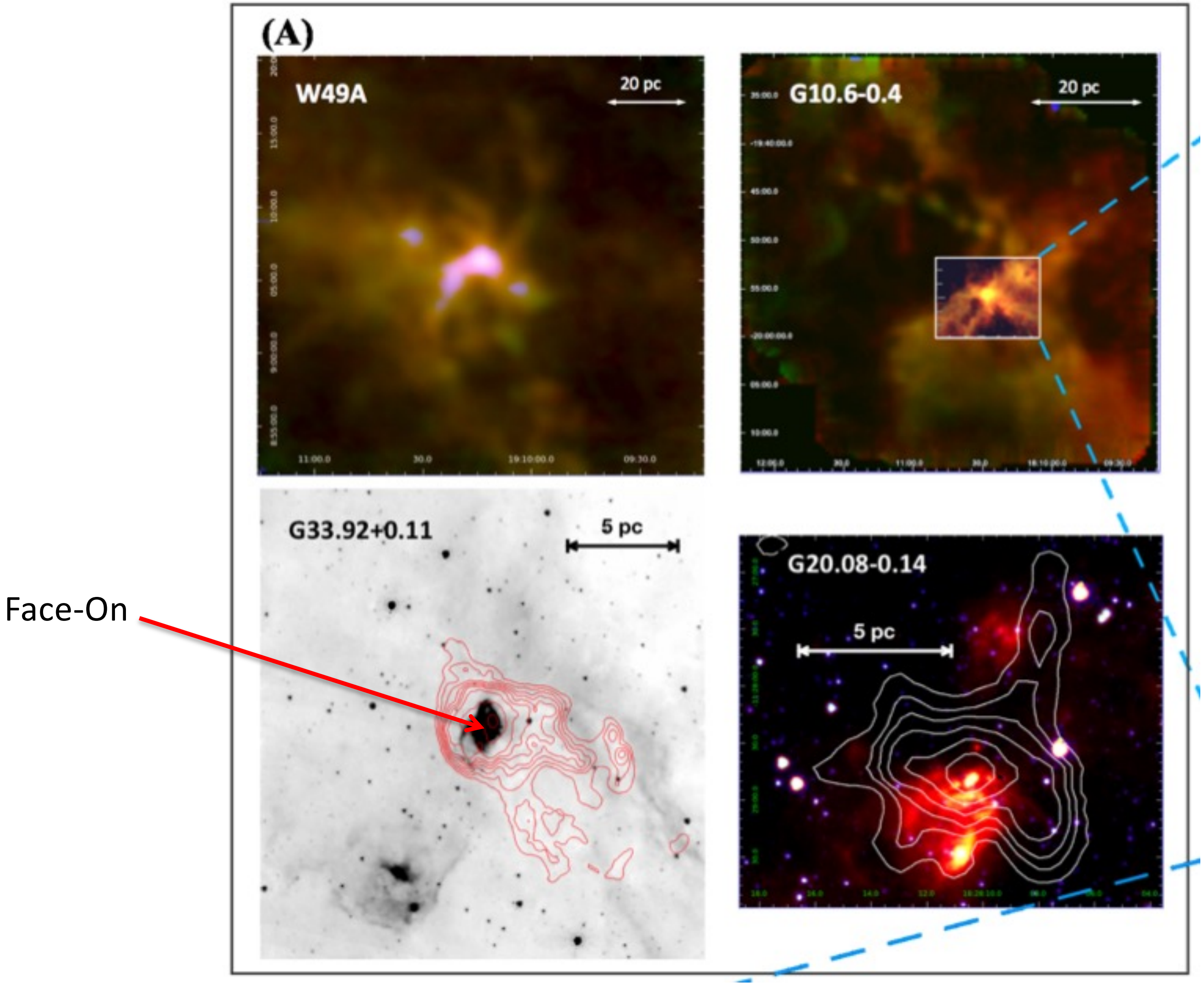
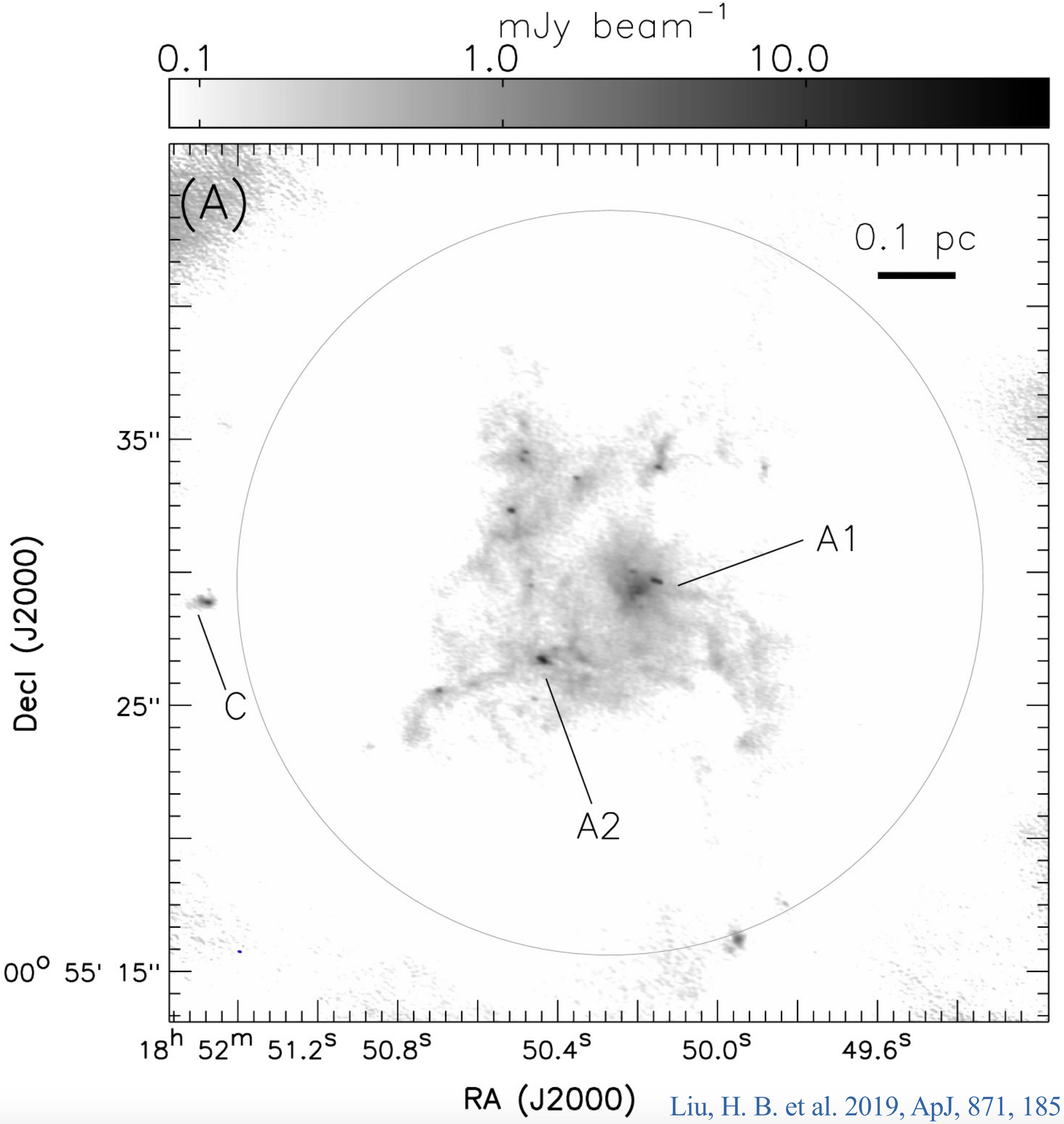


(D)
SMM 1a
SMM 1c
SMM 4
SMM 5b
SMM 2b
SMM 5a
SMM 1b
SMM 2a
SMM 3a
SMM 3b
0.1pc

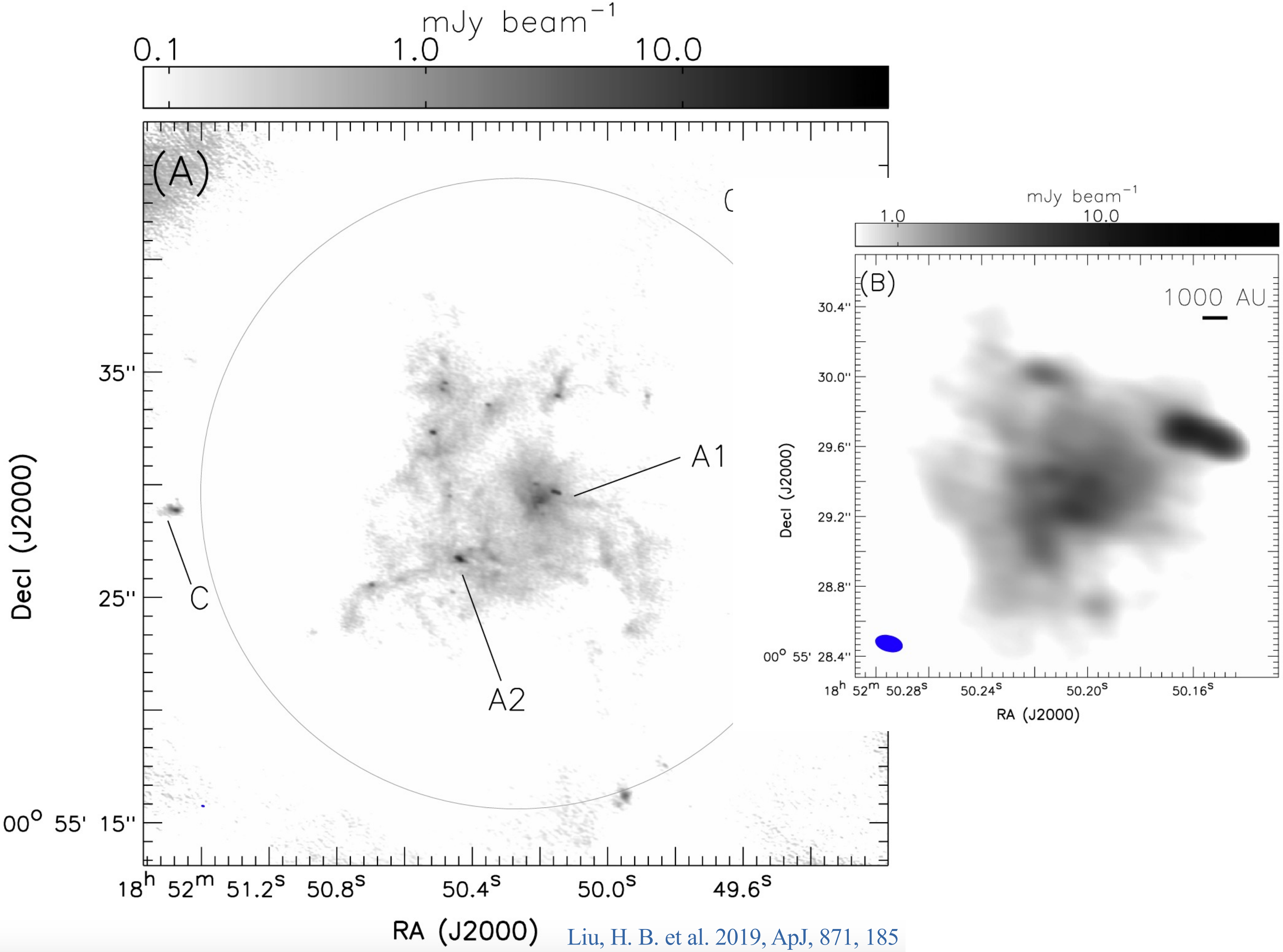

(A)
W49A
20 pc
G10.6-0.4
20 pc
G33.92+0.11
5 pc
Face-On
G20.08-0.14
5 pc

Flattened, gravitationally unstable rotating structure



Liu, H. B. et al. 2019, ApJ, 871, 185

Flattened, gravitationally unstable rotating structure



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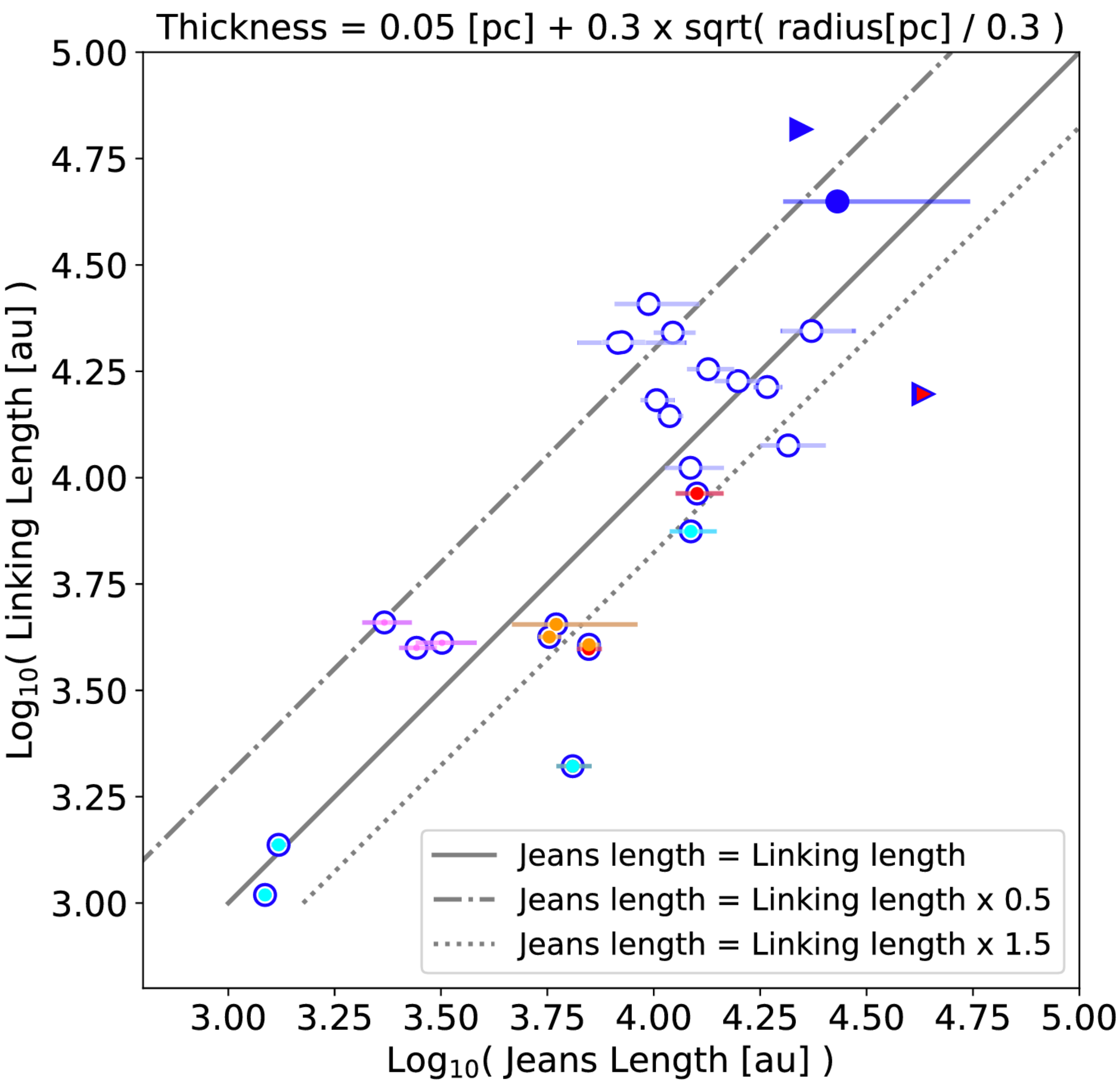

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Summary – What we do not understand

Kennicutt-Schmidt Law & Gao-Solomon Relation

Stellar Initial Mass Function

Energetic and Kinematics in the Star-forming Molecular Clouds

1. Role of turbulence
2. Role of magnetic field
3. Role of Feedback and cloud-cloud collision, galactic dynamics, etc.

The origin of SMBH and the formation of the M-σ relation